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The Generalized Theory of Transfers and Welfare: Bilateral Transfers in a Multilateral World

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The Generalized Theory of Transfers and Welfare: Bilateral Transfers in a Multilateral World

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Paul Samuelson's (1952, 1954) classic papers on the transfer problem addressed two separate analytical issues: the "positive" effect of a transfer on the terms of trade; and the welfare effect of the transfer on the donor and the recipient.

Since then, a considerable body of literature has grown up on the positive analysis. While Samuelson (1954) himself had extended the $2 \times 2 \times 2$ free trade analysis to allow for tariffs and transport costs, subsequent writers have analyzed other extensions of the model: for example, to allow for nontraded goods as with leisure in Samuelson (1971); or general nontraded goods in John Chipman (1974) and Ronald Jones (1970, 1975).

Remarkably, however, the welfare analysis of transfers has not paralleled these developments. Since Wassily Leontief (1936) produced an example of immiserizing transfer from abroad and Samuelson (1947) argued that the example required market instability, the proposition that has monopolized attention has been that a transfer in the conventional $2 \times 2 \times 2$ model in its free trade version cannot immiserize the recipient or enrich the donor as long as world markets are stable (in the Walras sense). Interestingly, Samuelson

(1954), who did extend the positive analysis to include tariffs, did not go on to ask whether immiserization of the transfer recipient (and hence symmetrically enrichment of the donor in a two-country model) could now arise consistent with market stability.

Recently, the welfare analysis of transfers has been extended in two different directions, both apparently unconnected, and both yielding the conclusion that transfers from abroad can be immiserizing (and that the donor may improve its welfare) despite market stability. One route to this conclusion has been the introduction of a third economic agent (or country) that is outside of the transfer process. In the Appendix of his 1960 paper analyzing the interaction between trade policy and income distribution, Harry Johnson discussed the possibility of welfare-paradoxical redistribution between two factor-income classes (capital and labor) in an open economy, thereby providing what can be interpreted as a treatment of the three-agent transfer problem for the case in which donor and recipient are both completely specialized in the ownership of a single different factor.[1] An independent analysis of the three-agent transfer problem, using a restrictive model with given endowments of goods and fixed coefficients in consumption, was also undertaken in an important paper by David Gale (1974).[2] Brecher and Bhagwati

[1] After the present paper was submitted for publication, and following its presentation at Rochester, our attention was drawn to this Appendix, which was noticed by a student of Ronald Jones. Subsequently, we learned from Makoto Yano that Motoshige Itoh had pointed out an important related paper by Ryuotaro Komiya and T. Shizuki (1967), whose condition (11) for the Johnson case anticipated our equation (12) below. We are grateful for having both of these references brought to our attention.

[2] Gale constructs an example in which the donor is enriched along with the recipient. Furthermore, this immediately implies that a reverse transfer will immis-

(1981) also independently pioneered this analysis in the context of a three-agent model where the recipient country is split into one subset of "national" factors and another of "foreign" factors, and the conditions for the immiserization of the national factors after receiving a transfer from abroad are analyzed and shown explicitly to be consistent with market stability.

Another route has been to consider transfers in the presence of exogenously specified domestic distortions. Thus, Brecher and Bhagwati (1982) have analyzed the case of a transfer in the presence of a production distortion in the recipient country and shown that the recipient can get immiserized despite market stability if the recipient's "overproduced" good is inferior in the donor's consumption. Hatta, in an early unpublished paper (1973a), has also demonstrated for a closed economy with constant-cost production that a transfer between two agents, when there is a distortionary wedge between producer and consumer prices, could immiserize the recipient consistent with market stability. Peter Diamond (1978) has also recently considered the welfare impact of transfers when a price distortion exists in an economy with convex technology, and he gives comparative-static results that are consistent with paradoxes.

This recent proliferation of paradoxical cases of immiserizing transfers (and enriching transfer payments) is reminiscent of the earlier multiplication of cases involving immiserizing growth, with Bhagwati's (1958) analysis of the case of a large country in free trade being followed by Harry Johnson's (1967) analysis of the case of a small country

with a tariff.[3] The latter proliferation led to the generalized theory of immiserizing growth (Bhagwati, 1968b) whose major, influential proposition is that growth, in the presence of a distortion implying departure from full optimality, can be immiserizing since the primary gain from growth at optimal policies may be outweighed by an accentuation of the loss from the distortion vis-à-vis the optimal policies.

Can a similar, striking generalization be developed in regard to the transfer-induced paradoxes? It is the general conclusion of our analysis in this paper that, indeed, it can. We demonstrate that the phenomenon of immiserizing transfers from abroad (and the analytically symmetric phenomenon of enriching transfer payments) in the presence of market stability can arise only if there is a distortion characterizing the economy in question.

This general conclusion is critically dependent on our demonstration below that the three-agent case, which appears prima facie to involve no distortion while producing the noted paradoxes, is indeed characterized by what Bhagwati (1971) has called a *foreign* distortion, since the country is not using an optimal tariff. Moreover, the exercise of their joint monopoly power by the recipient and donor (viewed as members of a customs union) vis-à-vis the nonparticipant agent will be shown to eliminate the paradoxes in question.

Thus, in Section I, we develop the basic analysis of transfers when there are two economic agents (countries) engaged in the transfer process, but there is an added agent *outside* the transfer process so that we have a bilateral transfer in a multilateral context. Conditions are established for immiserization of the recipient, for enrichment of the donor and for the "double perversity" when these two paradoxical outcomes arise simultaneously. Economically intuitive explanations of these results are derived in a number of alternative ways.

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erize the (new) recipient. A simple calculation, moreover, shows that the Gale example is Walras-stable. Gale's work has stimulated a number of papers, most of which assume fixed commodity endowments and/or fixed consumption coefficients. A notable exception is an analysis of the three-agent transfer problem by Makoto Yano (1981), who introduces substitutability in both production and consumption. (We are grateful to Ronald Jones and Peter Neary for drawing our attention to Yano's work, after the research for the present paper was virtually completed.) A fuller discussion of these and related papers, in an evidently growing and important literature, is provided by us (1982b) elsewhere.

[3] Robert Aumann and B. Peleg (1974) have rediscovered, in a restrictive model with no substitution in production, the immiserizing growth case of Bhagwati (1958). See also Bhagwati (1982).

In Section II, yet further intuition on these results, in consonance with the theory of immiserizing growth, is arrived at, and suitable geometry of the three-agent transfer problem is simultaneously developed. Importantly, the role of inferiority in consumption or inelastic foreign demand is established in making feasible the perverse outcomes, which are shown to involve a foreign distortion (correctable by a uniform optimal tariff policy applied jointly by the donor and the recipient against the nonparticipant). In turn, this establishes an interesting parallel between the conditions for the immiserizing-transfer paradox in the three-agent, foreign-distortion case and the conditions established in Bhagwati's (1958) immiserizing-growth case which also involves a similar foreign distortion (i.e., growth for a large country that is failing to use an optimal tariff because of its free trade policy).

Section III then presents the implications of our results for some important theoretical and policy problems in both international and closed-economy contexts.

I. Transfers with Three Agents: Model and Analysis

We begin with a formal analysis of the three-agent transfer problem, drawing on duality theory in terms of compensated demand functions, which have been introduced into the welfare-theoretic analysis of international trade by Hatta (1973b, 1977), Hatta and Takashi Fukushima (1979), and most notably and comprehensively by Avinash Dixit and V. Norman (1980), although earlier applications such as indirect utility functions are to be found also in the work of Chipman (1972).

A. *The Model*

Consider a world economy consisting of three countries: α, β, and γ. (While the analysis is couched in terms of three countries, it is applicable immediately to a closed-economy context with three agents within the economy, or to a two-country international economy where one country is disaggregated into two groups as in Brecher-Bhagwati,

1981.) Each country produces and consumes two goods, X and Y. Free trade and perfect competition prevail.

Now, suppose that country α makes a transfer to country γ. Country β does not participate in the transfer process. We will call α the *donor*, γ the *recipient*, and β the *nonparticipant* "outside" country. The objective of the analysis will be to determine the effect of the transfer on the welfare levels of the three countries.

The following notation will be used in presenting our model:

q = the relative price of good X,
u^i = the welfare level of country i,
T = the value of the transfer in terms of good Y,
$e^i(q, u^i)$ = the expenditure function of country i,
$r^i(q)$ = the revenue function of country i,
$x^i(q, u^i)$ = the compensated import-demand function for good X by country i, for $i = \alpha, \beta, \gamma$.

We then define an *overspending function* c^i as follows:

$$c^i(q, u^i) \equiv e^i(q, u^i) - r^i(q), \qquad i = \alpha, \beta, \gamma.$$

Evidently, the value of this overspending function represents the difference between the expenditure necessary to achieve the utility level u^i when the goods-price ratio is q *and* the revenue of the producers of country i at the same price ratio. Thus, c^i is the amount of added revenue (i.e., transfer income) that is necessary for this country to sustain u^i when the price ratio is q.

Using this notation, we can write our model as follows:

$$(1) \qquad c^\alpha(q, u^\alpha) + T = 0,$$

$$(2) \qquad c^\beta(q, u^\beta) = 0,$$

$$(3) \qquad c^\gamma(q, u^\gamma) - T = 0,$$

$$(4) \quad x^\alpha(q, u^\alpha) + x^\beta(q, u^\beta) + x^\gamma(q, u^\gamma) = 0.$$

This model of four equations contains four variables: u^α, u^β, u^γ, and q. Equations (1)–(3) are the budget equations for the respective countries, while equation (4) is the market

equilibrium condition for good X. (In view of Walras' Law, the market-clearing equation for good Y has been omitted.)

B. Comparative Statics

We now examine the impact of an exogenous increase in T upon the variables of the model above. Throughout the paper, subscripts always indicate partial differentiation with respect to a particular variable; for example, $c_u^\alpha \equiv \partial c^\alpha / \partial u^\alpha$ and $x_q^\gamma \equiv \partial x^\gamma / \partial q$. The following theorem can now be derived.

THEOREM 1: *Assume (without loss of generality) that* $e_u^\alpha = e_u^\beta = e_u^\gamma = 1$ *initially; and let* $\Delta \equiv x^\alpha x_u^\alpha + x^\beta x_u^\beta + x^\gamma x_u^\gamma - x_q$, *where* $x_q \equiv x_q^\alpha + x_q^\beta + x_q^\gamma$. *Then*

$$(5) \quad dq/dT = (x_u^\gamma - x_u^\alpha)\big/\Delta,$$

$$(6) \quad du^\alpha/dT = \big[x_q - x^\beta(x_u^\beta - x_u^\gamma)\big]\big/\Delta,$$

$$(7) \quad du^\beta/dT = -\big[x^\beta(x_u^\gamma - x_u^\alpha)\big]\big/\Delta,$$

$$(8) \quad du^\gamma/dT = -\big[x_q - x^\beta(x_u^\beta - x_u^\alpha)\big]\big/\Delta.$$

PROOF:

Taking the total differential of (1) through (4), applying the assumptions of the theorem, and using the well-known property that $c_q^i = x^i$ (for $i = \alpha, \beta, \gamma$), we obtain

$$
\begin{bmatrix}
1 & 0 & 0 & x^\alpha \\
0 & 1 & 0 & x^\beta \\
0 & 0 & 1 & x^\gamma \\
x_u^\alpha & x_u^\beta & x_u^\gamma & x_q
\end{bmatrix}
\begin{bmatrix}
du^\alpha \\
du^\beta \\
du^\gamma \\
dq
\end{bmatrix}
=
\begin{bmatrix}
-1 \\
0 \\
1 \\
0
\end{bmatrix}
dT.
$$

Applying Cramer's Rule to this system and taking notice of (4), we immediately obtain the theorem.

It is readily shown that Δ equals (minus) the slope of the general equilibrium, excess-demand schedule of good X for the world as a whole.[4] Thus, the Marshall-Lerner condi-

tion for (Walrasian) stability implies that $\Delta > 0$.

Note also that equations (5) and (7) yield $du^\beta/dT > 0$ if and only if $-x^\beta \, dq/dT > 0$. Thus, the welfare of the country not involved in the transfer improves if and only if the price of its export good goes up as a result of the transfer—as is indeed immediately evident.

C. Paradoxes: Enrichment of Donor and Immiserization of Recipient

The welfare impacts of a transfer upon the donor and the recipient, however, are not as simple as this. In the remainder of this section, therefore, we will give various interpretations of Theorem 1, to shed more light on the conditions under which the paradoxes of immiserized recipient and enriched donor arise. Note immediately, however, that if either $x^\beta = 0$ or $(x_u^\beta - x_u^\gamma) = 0$, that is, if either β's net trade is zero or β and γ share an identical marginal propensity to consume X,[5] the second term in the numerator of the right-hand side of (6) is zero. In this case, equation (6) reduces to $du^\alpha/dT = x_q/\Delta$, which is, of course, the familiar expression for the welfare effect on the donor in the two-country analysis. With $\Delta > 0$ and $x_q < 0$, du^α/dT must be negative, that is, the donor must be immiserized. When the only trade partner of the donor is the recipient, or when the recipient and the nonparticipant share an identical marginal propensity, therefore, the welfare impact on the donor is *as if* we were in a two-country world, and the donor paradox never arises. A symmetric conclusion can be derived for the welfare effect on the recipient from equation (8).

Generally, however, the second term in the numerator of the right-hand side of (6) or (8)

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[4] First use equations (1), (2), and (3) to write $u^\alpha = v^\alpha(q, T)$, $u^\beta = v^\beta(q)$ and $u^\gamma = v^\gamma(q, T)$, respectively. Then define $x^\alpha[q, v^\alpha(q, T)] + x^\beta[q, v^\beta(q)] +$

$x^\gamma[q, v^\gamma(q, T)] \equiv \tilde{x}(q, T)$, which is the world's uncompensated excess-demand function for good X. Now, we have $\tilde{x}_q = x_q^\alpha + x_u^\alpha v_q^\alpha + x_q^\beta + x_u^\beta v_q^\beta + x_q^\gamma + x_u^\gamma v_q^\gamma = -x^\alpha x_u^\alpha - x^\beta x_u^\beta - x^\gamma x_u^\gamma + x_q \equiv -\Delta$.

[5] As may be readily verified, the marginal propensity to consume good X in country ι equals qx_u^ι/e_u^ι for $i = \alpha, \beta, \gamma$. Therefore, if β and γ share an identical marginal propensity to consume X, this implies that $x_u^\alpha = x_u^\gamma$ (recalling the normalization that $e_u^\alpha = e_u^\beta = e_u^\gamma = 1$).

can cause paradoxical welfare effects, and we have the following necessary conditions for the paradoxes:

(9) $\quad \dfrac{du^\alpha}{dT} > 0$ implies that $x^\beta \left(x_u^\beta - x_u^\gamma \right) < 0,$

(10) $\quad \dfrac{du^\gamma}{dT} < 0$ implies that $x^\beta \left(x_u^\beta - x_u^\alpha \right) < 0.$

In fact, (6) and (8) make it clear that when $x_q = 0$ (i.e., when substitution effects are assumed away), the second inequalities in (9) and (10) are not merely necessary but also sufficient conditions for the paradoxes. And it is equally clear that if x_q is sufficiently negative (i.e., if X and Y are readily substitutable in production and consumption), the paradoxes are unlikely to occur.[6]

D. Decomposition of Welfare Changes

In further understanding our results in Theorem 1, note first that the right-hand sides of (6)–(8) contain the x_u^i terms for all countries i ($= \alpha, \beta, \gamma$), *except* for the one whose welfare is stated by the equation in question. Let us now examine why this curious fact holds; it leads us into an insightful way of looking at our results.[7]

For this purpose we may conceptually decompose the transfer from α to γ into two stages. At the *first* stage, α gives transfers to both β and γ in proportion to their initial import demand for X.[8] At the *second* stage, β

gives γ what it received from α in the first stage, with the final situation ending up therefore as equivalent to the actual transfer going exclusively from α to γ. The welfare effect on the donor α can then be decomposed into two effects corresponding to these two stages.

Rewriting (6), we have[9]

(11) $\quad du/dT = x_q/\Delta$

$$- \frac{x^\beta}{\left(x^\beta + x^\gamma \right)} \frac{\left(x^\beta + x^\gamma \right)\left(x_u^\beta - x_u^\gamma \right)}{\Delta}.$$

Now it is possible to show that the first stage leads to the first term on the right-hand side of (11).[10] Making transfers to every other country in the world economy in proportion to its initial import demand for X is therefore tantamount to making a transfer to the other country in a two-country context! This process therefore results in a negatively signed term; the paradox of donor enrichment cannot come from this stage.

On the other hand, the second stage leads to the second term on the right-hand side of (11).[11] The sign of this term depends exclu-

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[6] This might explain why Gale (1974), who tried to construct an example of donor enrichment, wound up assuming fixed coefficients in consumption (with fixed coefficients in production also implied by his exchange model), and confessed his inability to admit "smooth preferences." Interestingly, the absence of smooth preferences also characterizes the examples that Gale attributes to other major mathematical economists such as Drezè and McFadden. Just recently, Daniel Leonard and Richard Manning (1982) provided a paradoxical example involving smooth preferences within an exchange model.

[7] In Section II and in fn. 15 below, we spell out an alternative way of seeing why the income terms of only the two "other" countries appear in equations (6) and (8).

[8] That is, when α gives out a transfer of one unit of Y, β receives $x^\beta/(x^\beta + x^\gamma)$ units of Y and γ receives $x^\gamma/(x^\beta + x^\gamma)$ units of Y. If these ratios are positive,

both countries receive positive amounts of transfer. If $x^\beta/(x^\beta + x^\gamma)$ is negative, β receives a negative transfer, viz., it gives a transfer of $-x^\beta/(x^\beta + x^\gamma)$ to α. In this situation, γ receives one plus $-x^\beta/(x^\beta + x^\gamma)$ units of Y, since $x^\gamma/(x^\beta + x^\gamma) = 1 - x^\beta/(x^\beta + x^\gamma)$. Similarly, γ gives a positive transfer to α when $x^\gamma/(x^\beta + x^\gamma)$ is negative.

[9] The reader can similarly rewrite (8) for the welfare effect on the recipient, γ.

[10] To see this, appropriately exploit the essence of the result in (6). A unit transfer from α to β implies

(a) $\quad du^\alpha/dT^{\alpha\beta} = \left[x_q - x^\gamma \left(x_u^\gamma - x_u^\beta \right) \right]\Big/\Delta,$

and from α to γ implies

(b) $\quad du^\alpha/dT^{\alpha\gamma} = \left[x_q - x^\beta \left(x_u^\beta - x_u^\gamma \right) \right]\Big/\Delta.$

With actual transfers in the first stage divided according to the ratios $x^\beta/(x^\beta + x^\gamma)$ and $x^\gamma/(x^\beta + x^\gamma)$ between β and γ, these ratios should be multiplied into (a) and (b), respectively. Adding the resulting equations yields x_q/Δ.

[11] This readily follows from appropriately using the essence of the result in (7) and the fact that $x^\beta/(x^\beta + x^\gamma)$ represents the share of the transfer β received at the first stage and hands out to γ in the second stage. Equation

sively on the direction of the price change caused by the second-stage transfer. We already know from (9) that $x^\beta(x_u^\beta - x_u^\gamma) < 0$ is a necessary condition for the paradox of donor enrichment, and why this occurs is readily seen from (11) and the second-stage argumentation.

E. Alternative Necessary Conditions for Paradoxes

We now turn to an alternative, equally insightful way of looking at Theorem 1. We first establish a set of necessary conditions for the paradoxes of donor enrichment and recipient immiserization. Then, it will be shown how these conditions are also necessary for price amplification effects which further help to explain the paradoxical possibilities.

Take again the case of donor welfare, and apply the Slutsky equation to (6) to get[12]

$$(12) \quad \frac{du^\alpha}{dT} = \frac{\left(x_q^\alpha + x_q^\gamma + \tilde{x}_q^\beta + x^\beta x_u^\gamma \right)}{\Delta},$$

where $\tilde{x}^\beta(q)$ is the uncompensated import-demand function for country β. Now, given $\Delta > 0$, and assuming throughout the rest of this section without loss of generality that $x^\beta < 0$ (i.e., country β exports good X), we see immediately that the donor can be enriched only if either $x_u^\gamma < 0$, or $\tilde{x}_q^\beta > 0$, or both. That is, if a transfer enriches donor α, then either X is an inferior good to the recipient γ or the offer curve of the nonparticipant outside country β is inelastic (such that the export supply of X by β falls as the relative price of X rises).

Similarly, for the immiserization of the recipient, we must have $du^\gamma/dT < 0$, and this

can be shown to imply that either $x_u^\alpha < 0$ or $\tilde{x}_q^\beta > 0$ or both.

To understand more fully why these conditions are necessary for the paradox of (say) donor enrichment, the Slutsky equation and (4) may be used straightforwardly to rewrite the stability condition as $\Delta = x^\alpha(x_u^\alpha - x_u^\gamma) - x_q^\alpha - x_q^\gamma - (\tilde{x}_q^\beta + x^\beta x_u^\gamma) > 0$. If $(\tilde{x}_q^\beta + x^\beta x_u^\gamma) > 0$—which can happen only if either $\tilde{x}_q^\beta > 0$ or $x_u^\gamma < 0$ (given still that $x^\beta < 0$)—Δ will be smaller than in the two-country case (in which $\tilde{x}_q^\beta = x^\beta = 0$), ceteris paribus. Therefore, the price change measured by equation (5) is amplified by the presence of the third (nonparticipant) country β. If this price-amplification effect applies to an improvement in the terms of trade for α, the donor may be paradoxically enriched by the transfer, even though the (smaller) terms-of-trade improvement in the two-country case cannot be great enough for the paradox of donor enrichment. By similar reasoning,[13] if β's offer curve is inelastic or good X is inferior for α, an amplified deterioration in γ's terms of trade may be great enough for the paradox of recipient immiserization.

These necessary conditions for an international transfer paradox are, interestingly, analogous to those established by Bhagwati (1958) for immiserization due to domestic growth (in the form of factor-endowment expansion or technological improvement). As he showed, the paradoxical possibility of immiserizing growth requires that either growth be ultra-biased against production of the importable (i.e., the importable be an "inferior" good in production) or the foreign offer curve be inelastic. This analogy suggests immediately that, if immiserizing growth paradoxes are attributable to the presence of distortions, as shown in Bhagwati (1968b), it should be possible to interpret the present transfer analysis in the three-agent context also as one where the paradoxes of immiserized recipient and enriched donor arise only when a distortion is present. The distortion

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(7) applies since, when β makes a transfer to γ, the welfare effect on α is as if α is the nonparticipant, outside country; the resulting welfare impact on α per unit transfer from β to γ is

$$\frac{du^\alpha}{dT^{\beta\gamma}} = -\frac{x^\alpha\left(x_u^\gamma - x_u^\beta\right)}{\Delta} = -\frac{\left(x^\beta + x^\gamma\right)\left(x_u^\beta - x_u^\gamma\right)}{\Delta}.$$

[12]Substituting $\tilde{x}_q^\beta = x_q^\beta - x^\beta x_u^\beta$ into (6) and recalling $x_q = x_q^\alpha + x_q^\beta + x_q^\gamma$ yields (12).

[13]In this case, the Slutsky equation and (4) should be used to rewrite the stability condition as

$$\Delta = x^\gamma\left(x_u^\gamma - x_u^\alpha\right) - x_q^\alpha - x_q^\gamma - \tilde{x}_q^\beta - x_q^\beta x_u^\alpha > 0.$$

here, as in Bhagwati (1958), must again arise as a foreign distortion in the sense of Bhagwati (1971); that is, the failure to exploit monopoly power in trade. Indeed, this can be demonstrated, as in Section II below.

II. Viewing Three-Agent Paradoxes as Resulting from Foreign Distortion

We now proceed to demonstrate that the perverse welfare responses to bilateral transfers in the multilateral framework of three agents are attributable to the presence of a foreign distortion, and that the introduction of a suitable optimal tariff that eliminates this distortion will rule out the paradoxes. We first demonstrate this geometrically, using a technique that is suitable for "large" (as well as "small") transfers.

A. The Geometry of the Free Trade Case

We begin by illustrating in Figure 1 the possibility of a perverse welfare response to bilateral transfer in the three-agent case. For convenience of exposition without loss of generality, the diagram treats countries α and γ as partners of a customs union engaged in (free) trade with country β. (This treatment takes on more than expositional importance in Part B below, when α and γ uniformly impose an optimal tariff policy against β.) In the initial pretransfer equilibrium, the union produces on its production-possibility frontier $Q_y Q_x$ at point Q, consumes on its Scitovsky (1942) frontier $S_y S_x$ at point S, and trades with country β from point Q to point S along the price line QS.[14] To avoid cluttering the diagram, we have not drawn country β's offer curve, which starts at point Q and passes through point S. For the sake of concreteness only, let country β be again an exporter of commodity X (i.e., $x^{\beta} < 0$) while country α imports this good (with $x^{\alpha} > 0$).

Now, with country α making a transfer to country γ, suppose that the former's terms of
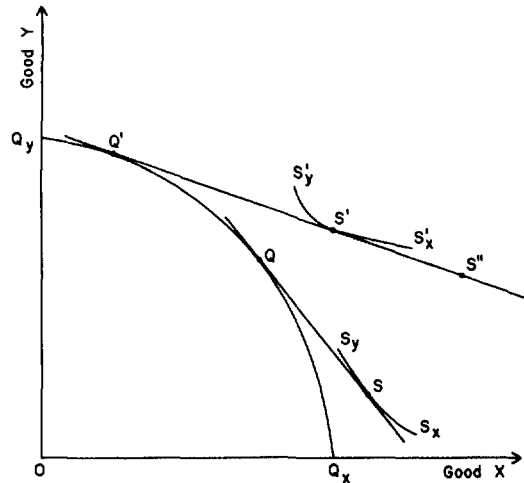

FIGURE 1

trade consequently improve because the marginal propensity to consume good X is greater for country α than for its union partner. Figure 1 illustrates the borderline case in which the terms-of-trade improvement is exactly enough to leave country α's welfare unchanged, despite the transfer. The union shifts in production to point Q' on curve $Q_y Q_x$, and moves in consumption to point S' on curve $S_y' S_x'$, which is another Scitovsky frontier in the map corresponding to a constant level of country α's welfare. Country β's offer curve (still not drawn) now starts at point Q' and passes through point S'.

In Figure 1 as drawn, good X is clearly inferior for the union as a whole. This inferiority, moreover, must characterize country γ in particular, since country α's welfare is constant throughout the entire Scitovsky map. By contrast, no such inferiority would be implied if curve $S_y' S_x'$ were redrawn to touch line $Q'S'$ at point S'' (lying east of point S), while country β's offer curve (not shown) were redrawn to pass through point S'' when starting at point Q'. In this alternative case, however, the offer curve of country β must be inelastic, because a deterioration in this country's terms of trade is now associated with a rise in exports to the union. (These two alternative conditions are, of course, those already established in Section I,

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[14] For further details on the use of production-possibility and Scitovsky frontiers corresponding to a pair of countries involved directly in a bilateral transfer, see Brecher and Bhagwati (1982).

Part E, above for the paradox of donor enrichment.)

By similar reasoning, an actual rise in country α's welfare, in response to a transfer to country γ, might occur provided that *either* good X is inferior in the latter country *or* the offer curve of country β is inelastic. This paradoxical possibility would occur in Figure 1 if country β's offer curve (not drawn) were respecified to cross line $Q'S'$ southeast of the consumption point S' (or alternatively S'') after starting at the production point Q'. In this case, after the transfer, there would remain a world excess supply of good X if q fell only enough to leave u^α constant at the initial (pretransfer) level. Thus, given stability, country α's terms of trade would ultimately have to improve still further, thereby leading to the paradox of donor enrichment.[15]

Essentially the same argument shows that country γ might incur a welfare loss from

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[15]In determining whether there remains a world excess supply of good X when q is adjusted to keep u^α constant after the transfer, clearly the substitution effect but not the income effect plays a role in (the unchanged-welfare) country α, whereas both of these effects are relevant in countries β (as u^β varies with q) and γ (as u^γ varies between Scitovsky frontiers). Thus, we have additional insight into why the income effects x_u^β and x_u^γ but not x_u^α enter the necessary condition (9). Similar reasoning sheds extra light on (10)

Two further remarks are in order. First, the use of the Scitovsky technique to analyze *welfare* changes here is only a natural counterpart to the two-stage derivation technique used by trade theorists in *positive* analyses. Thus, if the impact of a parametric change on the terms of trade is analyzed, one can hold the terms of trade constant, compute excess demand for one of the two goods, and then use the stability condition to determine the direction of the terms-of-trade change, a procedure introduced by Johnson, Mundell, and many other trade theorists in the 1950's. Identically, if one is interested instead in welfare change (of say the donor), one can equally hold welfare (of the donor) constant, compute excess demand, use the stability condition to determine the terms-of-trade change and then immediately the welfare change. This was, in fact, the procedure utilized in the analysis of immiserizing growth in Bhagwati (1958); and for the transfer problem, it leads naturally to the resurrection of the Scitovsky curves, as in the text above. Second, the use of Scitovsky curves in analyzing the transfer paradoxes was introduced earlier in Brecher and Bhagwati (1982). Whereas our Scitovsky technique easily handles large changes, the use of the two-stage holding-welfare-constant technique for small changes has also been introduced in Yano (1981) who credits Ronald Jones for the idea.

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receiving the transfer, provided that either good X is inferior in country α, or (as before) the offer curve of country β is inelastic. If the required conditions for a perverse response in welfare hold simultaneously for both partners of the union, the transfer from country α to country γ could raise the former's welfare and lower the latter's, implying a double perversity of outcomes.

B. *Optimal Tariff Against a Nonparticipant Country*

Consider now the following alternative ways of demonstrating how the use of an optimal tariff by the union rules out the paradoxes at issue.

1. *A Geometric Analysis.* Consider the extension of the preceding analysis to the case where the union of α and γ always maintains a uniform, optimal tariff vis-à-vis β, the nonparticipant country. Thus, for each value of the domestic goods-price ratio in Figure 2, the union adjusts the tariff to set the world-price ratio at the level consistent with the Robert Baldwin (1948) envelope BE, given the union's production-possibility frontier Q_yQ_x and the offer curve QG of country β. To avoid cluttering the diagram, we have drawn this offer curve with its origin in only one of the many possible positions. Alternatively, if this origin were placed at point Q' (instead of Q) for example, the offer curve would touch curve BE at point S' (rather than S). Following a common convention, tariff revenues collected by each union member are returned to its consumers in lump sum fashion.

This optimal-tariff policy must result in the union consuming along its Baldwin envelope BE of Figure 2 in equilibrium. Initially, the union produces on its production-possibility frontier Q_yQ_x at point Q, consumes on its Scitovsky frontier S_yS_x at point S, trades with country β along the external-price line QS from point Q to point S, and imposes a tariff to create the proportional wedge between this price line and the (parallel) domestic-price lines (not drawn) tangent to curve Q_yQ_x at point Q and to curve S_yS_x at point S.
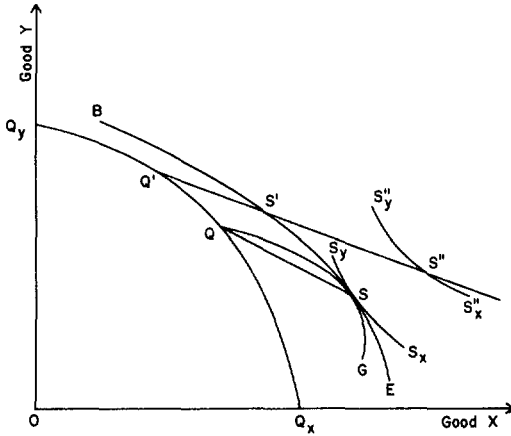
FIGURE 2

Now let the transfer take place from country α to country γ, and imagine what would hypothetically happen if the domestic relative price of good X within the union fell exactly enough to leave the donor country α's welfare constant, despite the transfer. Under these circumstances, the union would move to point Q' in production, trade along the external-price line $Q'S''$ with the rest of the world, and plan to consume at point S'' on the Scitovsky frontier $S_y''S_x''$ drawn for the initial level of country α's welfare. (For well-known reasons, the slope of curve $S_y''S_x''$ at point S'' equals the union's internal product-price ratio, given by the common slope of curves Q_yQ_x and BE at points Q' and S', respectively.) In this way, an excess demand (represented by the length $S'S''$) for good X for the world as a whole would necessarily emerge,[16] and the relative price of this good would have to rise to clear world markets under stable conditions.

This implies that country α, the donor, cannot enjoy enrichment since the initial fall in the price of X which exactly offset the primary loss from the transfer would now be reduced, leaving α worse off. Similarly, we

could establish that the recipient, γ, cannot be immiserized.

It follows, therefore, that the paradoxes of donor enrichment and recipient immiserization cannot arise if the union of the donor and the recipient follows the policy of adopting an optimal tariff that equates their domestic rates of (producer) transformation and (consumer) substitution to the foreign rate of transformation, such that $DRS^\alpha = DRS^\gamma = DRT^\alpha = DRT^\gamma = FRT$. Under free trade, however, the paradoxes become possible since the situation suffers from a foreign distortion such that $DRS^\alpha = DRS^\gamma = DRT^\alpha = DRT^\gamma \neq FRT$.

2. *Algebraic Analysis*: The preceding geometric analysis immediately suggests an approach to a formal proof of the proposition that the union of α and γ, utilizing an optimal tariff against β, would not admit of the paradoxes in question. Thus we should be able to show that, if such a tariff were in place, the paradoxes would be ruled out. This can indeed be done as follows.

Utilizing the model so far, we now distinguish between q as the domestic relative price of X and p as the external price. Then, we write the foreign offer-curve function as $\tilde{x}^\beta(p)$. Now, define the function ρ by

$$(13) \quad \rho(q, p) \equiv (q - p)\left[-\tilde{x}^\beta(p)\right].$$

If q and p take on their equilibrium values, then $\rho(q, p)$ gives the tariff revenue of the union of α and γ. Also, let $p^*(q)$ represent the value of p that maximizes $\rho(q, p)$, given q, and define the function ρ^* by

$$(14) \quad \rho^*(q) \equiv \rho[q, p^*(q)].$$

Now, if the international market is in equilibrium with $p = p^*(q)$, so that $\rho(q, p)$ takes on its maximized value $\rho^*(q)$ for the prevailing q, then it can be readily observed that the union must be operating on its Baldwin envelope at the point where FRT equals that q. In this case, therefore, $q - p^*(q)$ is equal to the union's optimal tariff rate. Of course, there is no a priori guarantee that any arbitrary q and the corresponding $p^*(q)$ are equilibrium values; in general, they are not. But

[16]Note that point S'' in consumption must lie outside the curve BE, assuming that good Y is not sufficiently inferior to violate the Vanek (1965)-Bhagwati (1968a)-Kemp (1968) condition (discussed in more detail by us elsewhere, 1982a) for stability in the presence of tariffs. See also fn. 17 below.

the observation above implies that if the union always sets the tariff rate equal to $q - p^*(q)$ for the prevailing q, then this rate coincides with the optimal tariff rate when equilibrium is reached. The following result can now be established.

LEMMA 1: *If* $p = p^*(q)$, *then* $(q - p)\tilde{x}_p^\beta = \tilde{x}^\beta$.

PROOF:

Recalling that the optimal tariff maximizes ρ given q, we have $\partial \rho[q, p^*(q)]/\partial p = 0$, from which the lemma follows immediately.

Now, reformulate the model of this paper to allow for the union always imposing the tariff rate $q - p^*(q)$. Also assume that countries α and γ collect tariff revenues equal to $[q - p^*(q)]x^\alpha(q, u^\alpha)$ and $[q - p^*(q)] x^\gamma(q, u^\gamma)$, respectively. (Thus, we implicitly assume that both union members import good X from country β, although the results of the analysis would be essentially unaffected if one member received all of the tariff revenues because the partner imported nothing from β.) The overspending functions c^α and c^γ are now given by

$$(15) \quad c^\alpha(q, u^\alpha) \equiv e^\alpha(q, u^\alpha) - r^\alpha(q)$$
$$- [q - p^*(q)] x^\alpha(q, u^\alpha),$$

$$(16) \quad c^\gamma(q, u^\gamma) \equiv e^\gamma(q, u^\gamma) - r^\gamma(q)$$
$$- [q - p^*(q)] x^\gamma(q, u^\gamma).$$

Then, our full revised model is given by

$$(17) \quad c^\alpha(q, u^\alpha) + T = 0,$$

$$(18) \quad c^\gamma(q, u^\gamma) - T = 0,$$

$$(19) \quad x^\alpha(q, u^\alpha) + \tilde{x}^\beta[p^*(q)]$$
$$+ x^\gamma(q, u^\gamma) = 0.$$

This three-equation model has three variables: u^α, u^γ and q.

The following theorem can be derived.

THEOREM 2: *If* (17)–(19) *hold and* (*by normalization*) *initially* $e_u^\alpha = e_u^\gamma = 1$, *then*

$$(20) \quad dq/dT = (x_u^\gamma - x_u^\alpha)/\Delta^t c_u^\alpha c_u^\gamma,$$

$$(21) \quad du^\alpha/dT = x_q^t/\Delta^t c_u^\alpha c_u^\gamma,$$

$$(22) \quad du^\gamma/dT = - x_q^t/\Delta^t c_u^\alpha c_u^\gamma,$$

where $\Delta^t \equiv - x_q^t + (c_q^\alpha x_u^\alpha/c_u^\alpha) + (c_q^\gamma x_u^\gamma/c_u^\gamma)$ *and* $x_q^t \equiv x_q^\alpha + \tilde{x}_p^\beta p_q^* + x_q^\gamma$.

PROOF:

Taking the total differential of (17)–(19), we get

$$
\begin{bmatrix} c_u^\alpha & 0 & c_q^\alpha \\ 0 & c_u^\gamma & c_q^\gamma \\ x_u^\alpha & x_u^\gamma & x_q^t \end{bmatrix}
\begin{bmatrix} du^\alpha \\ du^\gamma \\ dq \end{bmatrix} =
\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} dT.
$$

We thus obtain

$$(23) \quad \frac{du^\alpha}{dT} = \frac{\left[c_u^\gamma x_q^t - (c_q^\alpha + c_q^\gamma) x_u^\gamma \right]}{\Delta^t c_u^\alpha c_u^\gamma}.$$

From (15) and (16), we get

$$c_q^\alpha + c_q^\gamma = p_q^*(x^\alpha + x^\gamma) - (q - p^*)(x_q^\alpha + x_q^\gamma),$$

and

$$c_u^\gamma = 1 - (q - p^*)x_u^\gamma.$$

Thus, we have

$$(24) \quad \left[c_u^\gamma x_q^t - (c_q^\alpha + c_q^\gamma) x_u^\gamma \right]$$
$$= \left[x_q^t - (q - p^*)x_u^\gamma(x_q^\alpha + \tilde{x}_p^\beta p_q^* + x_q^\gamma) \right]$$
$$- \left[p_q^*(x^\alpha + x^\gamma) - (q - p^*)(x_q^\alpha + x_q^\gamma) \right] x_u^\gamma$$
$$= x_q^t - x_u^\gamma p_q^* \left[(q - p^*)\tilde{x}_p^\beta - x^\beta \right] = x_q^t,$$

where the last expression follows from Lemma 1 and equation (19). Substituting (24) into (23) immediately yields (21). The other equations in Theorem 2 are derived similarly.

It is evident then from (21) and (22) that, with $\Delta' c_u^\alpha c_u^\gamma > 0$ owing to market stability,[17] and with $x_q^t < 0$,[18] we necessarily get $du^\alpha/dT < 0$ and $du^\gamma/dT > 0$. That is, the donor must be immiserized and the recipient must be enriched. Paradoxes cannot arise.

3. *A General Proposition.* Now that we have demonstrated that the pursuit of an optimal tariff policy by the (union of the) donor and recipient jointly vis-à-vis the nonparticipant agent will rule out transfer paradoxes, in a Walras-stable context, we are able to see that the presence of a *suitably interpreted* (foreign) distortion is required in the three-agent case if the paradoxes are to arise.[19] At the same time, for the case of two agents, we know that *exogenously imposed* price distortions (for example, tax-cum-subsidies on production, consumption or trade) can also generate the transfer paradoxes (in the presence of inferior goods), as established by Brecher and Bhagwati (1982) and ourselves (1982a). We also know from the former paper, which analyzes transfer-induced distortions in the context of additionality requirements, and from the latter paper which analyses transfer-seeking *DUP* activities by domestic and foreign lobbyists, that *endogenous* (i.e., transfer-induced) distortions can also generate transfer paradoxes, consistent with Walrasian stability. We can therefore now state the following general proposition:

PROPOSITION: *The paradoxes of enriched donor and immiserized recipient cannot arise unless a distortion is present in the system.*

III. Conclusion: Implications for Analytical and Policy Problems

The foregoing analysis has important implications in a number of areas of theoretical and policy concern.

International: (*i*) Our analysis of the three-agent problem does modify the earlier theoretical presumption against the possibility of stability-compatible paradoxes. (*ii*) Since, in the international context, reparations and aid are never given by one country to the "rest of the world," but are always bilateral transactions in a multilateral context, policymakers should be alert to the possibility that their intentions may be frustrated by paradoxical outcomes. (*iii*) As noted by Brecher and Bhagwati (1981), the three-agent transfer problem has an immediate counterpart in the analysis of customs unions with full mobility of factors within the union. Thus, for instance, it is possible for Italy to be immiserized within the EEC by receiving an aid inflow from the non-EEC world, under conditions established by us, consistent with market stability.

Domestic: (*i*) Internal redistribution from the rich to the poor may also be counterproductive under the conditions established here. Thus, if the poor receive the transfer from the rich while the not-so-poor outside group is a net exporter of food and the rich also have a lower marginal propensity to consume food than the not-so-poor, then we know that the conditions are satisfied to make it possible for the poor to be immiserized by receipt of the transfer. (*ii*) The three-agent analysis also brings into sharp focus problems raised by the "basic needs" prescription that the targeted poor be given purchasing power to buy their nourishment et al. If this purchasing power is taken from the rich, the nonparticipant not-so-poor may well find that their real income is diminished by a transfer-induced deterioration in their terms of trade (under an appropriate ranking of marginal propensities to consume), so that the poor become not-so-poor

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[17]By reasoning similar to that of fn. 4 above, $\Delta' = \tilde{x}_q^t$, where $\tilde{x}^t(q, T) \equiv x^\alpha[q, \nu^\alpha(q, T)] + \tilde{x}^\beta[p^*(q)] + x^\gamma[q, \nu^\gamma(q, T)]$, while the indirect utility functions ν^α and ν^γ now come respectively from equations (17) and (18). Thus, Walrasian stability requires that $0 < \Delta'$. Also, as explained by us elsewhere (1982a), c_u^α and c_u^γ must both be positive to satisfy the Vanek-Bhagwati-Kemp condition assumed in fn. 16 above.

[18]As we move along the Baldwin envelope, $p_q^* > 0$ and $\tilde{x}_p^\beta < 0$ are well-known properties of economic efficiency. Thus, $x_q^\alpha + \tilde{x}_p^\beta p_q^* + x_q^\gamma < 0$.

[19]While our analysis can be viewed therefore as essentially providing a *conceptual* way of integrating the many-agent problem into the theory of distortions and welfare, our result on optimal tariffs also has direct *policy* relevance. For example, if there are two income classes, rich and poor, in an open economy with monopoly power in trade, we can definitely assert now that, if an optimal tariff is always in place, redistribution from the rich to the poor will not lead to paradoxical results.

whereas the not-so-poor are reduced to the ranks of the poor! Indeed, our three-agent analysis similarly implies a certain caution in treating famine relief through transfers of purchasing power to the distressed income groups. Unless a similar security net is available elsewhere, you may then be pushing the malnourished not-so-obviously-starving poor (who are not receiving this purchasing power) below the line so that *they* are now ravaged by the famine.

The Invisible Shakedown: Our analysis also suggests a generalization of the idea underlying Gale's (1974) example where both the donor and the recipient are enriched by a transfer, at the expense of the nonparticipant outside agent. What is implied here is a seemingly innocuous process that involves enrichment at the expense of an unsuspecting agent. Through this process, the outside agent is hurt, for the benefit of the transfer-process agents, in a fashion that is by no means perceived as such, unlike in overt and visible instances such as where an optimal tariff may be levied against that agent.

Gale's example is, however, only one such instance: where the transfer is between the two agents (α and γ) with the third agent (β) remaining outside of the transfer process. But it is easy to see that one of the two agents (say α) could equally exploit the third agent (β) by making a *direct* transfer to it, immiserizing it while enriching itself (and even, if need be, the other agent, γ): the conditions for this being readily established from equations (6)–(8) above. This is a clear case where a gift horse does need to be looked at in the mouth since, to mix metaphors ever so slightly, it turns out to be a Trojan horse.

The class of cases where (seemingly innocuous) transfers can improve the donor's welfare at the expense of either the direct recipient or an agent outside of the transfer process, or both, may then be christened generically as phenomena involving an *Invisible Shakedown*.

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