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Liability Design for Autonomous Vehicles and Human-Driven Vehicles: A Hierarchical Game-Theoretic Approach

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Abstract

Autonomous vehicles (AVs) are inevitably entering our lives with potential benefits for improved traffic safety, mobility, and accessibility. However, AVs’ benefits also introduce a serious potential challenge, in the form of complex interactions with human-driven vehicles (HVs). The emergence of AVs introduces uncertainty in the behavior of human actors and in the impact of the AV manufacturer on autonomous driving design. This paper thus aims to investigate how AVs affect road safety and to design socially optimal liability rules in comparative negligence for AVs and human drivers. A unified game is developed, including a Nash game between human drivers, a Stackelberg game between the AV manufacturer and HVs, and a Stackelberg game between the law maker and other users. We also establish the existence and uniqueness of the equilibrium of the game. The game is then simulated with numerical examples to investigate the emergence of human drivers’ moral hazard, the AV manufacturer’s role in traffic safety, and the law maker’s role in liability design. Our findings demonstrate that human drivers could develop moral hazard if they perceive their road environment has become safer and an optimal liability rule design is crucial to improve social welfare with advanced transportation technologies. More generally, the game-theoretic model developed in this paper provides an analytical tool to assist policy-makers in AV policymaking and hopefully mitigate uncertainty in the existing regulation landscape about AV technologies.

Keywords: Comparative Negligence Liability, Mixed Traffic, Hierarchical Game

1. Introduction

1.1. Motivation

Already an acknowledged transportation “game changer”, autonomous vehicles (AVs) are projected to arrive on public roads over the course of the next decade and disrupt the landscape of transportation ecosystems (Fagnant and Kockelman, 2015). Despite potential benefits for improved traffic safety, mobility, emissions, and accessibility (Wadud et al., 2016; Talebpour and Mahmassani, 2016; Fagnant and Kockelman, 2015; Chen et al., 2017; Kröger et al., 2019), the first-of-its-kind traffic fatality in Tempe, Arizona (Wakabayashi, 2018; Hawkins, 2019) involving a self-driving automobile also elicited tremendous attention among the public and policy makers about who should be liable in the interaction between autonomous vehicles and human drivers, cyclists and pedestrians. There are several possibilities, ranging from the human operator to prevailing conditions to the behavior of the pedestrian. From a legal perspective, these factual challenges are all but inevitable: Under Arizona law (and that of many other states), legal liability for accidents between automobiles and pedestrians typically involves a complex calculus of “comparative fault” assessments for each of the aforementioned groups. The involvement of an autonomous vehicle can complicate matters further...
by adding other parties to the mix, such as the manufacturers of hardware and programmers of software. Insurance coverage distorts matters further by including third party insurers to the mix. According to a letter issued by the Yavapai County Attorney (Polk), “there is no basis for criminal liability for the Uber corporation”, and the accident is now under further investigation by National Transportation Safety Board (NTSB) (NTSB, 2018). An article in Bloomberg (Beene and Levin, 2019) indicated that no cause has been assigned to this crash based on a recently released NTSB report. Such uncertainty in accident laws in the presence of AVs urges a prudently designed legal and policy system in place. In this paper, we seek to understand the research question: In an accident involving AVs, what an efficient liability rule is to apportion loss between road users so that the total social cost is minimized? A liability rule is “efficient” when both the injurer and the victim executes optimal care levels that minimizes a total social cost (Jain and Singh, 2002). Negligence-based liability is shown to be both necessary and sufficient to ensure an efficient liability rule (Jain and Singh, 2002), thus will be the focus of this paper.

As the ecosystem of motorized transportation transitions from human- to autonomously-driven vehicles, many transportation and legal experts anticipate that the underlying role of law and regulation will play a pivotal role in mediating the myriad local interactions within that ecosystem (Marchau et al., 2019). Autonomous vehicle policies cover a broad spectrum, ranging from cybersecurity, privacy, to vehicle licensing and land use. Accordingly, recent years have seen a growing body of literature on identifying and addressing challenges brought forth by AVs in regulatory regimes (Shladover and Nowakowski, 2017), urban planning policies (Fraedrich et al., 2019), and social interaction protocol design (Straub and Schaefer, 2019). However, one of the biggest challenges is that, unlike manufacturers of conventional human-driven vehicles (HVs), AV manufacturers can directly influence traffic by programming driving algorithms, making manufacturers an indispensable actor to be modeled in the new ecosystem. The changing composition of human and autonomous vehicles makes pressing the need to apportion liability risk among accident victims and the businesses who manufacture, design and market AVs (Marchant and Lindor, 2012; Geistfeld, 2017; Smith, 2017). However, research that quantifies the economic efficiency of liability rules for AVs is still in its nascent stage (Chatterjee and Davis, 2013; Chatterjee, 2016; Friedman and Talley, 2019; Shavell, 2019). Shavell (2019) argued for what is effectively a Pigouvian tax on accidents in environments where AVs completely saturate the traffic ecosystem (e.g., at some future date when AVs have fully penetrated and dominate the market). Because this scenario is unlikely to unfold in the near term (or perhaps ever, when one considers the presence of pedestrians, cyclists, and other human traffic actors), we concentrate instead on mixed AV-HV environment as a more immediately interesting case. Like us, Chatterjee and Davis (2013); Chatterjee (2016); Friedman and Talley (2019) also explored these mixed environments, analyzing how varying the legal standard associated with negligence and contributory / comparative negligence can distort human drivers’ interaction with AVs. Moreover, Friedman and Talley (2019) analyzed how various liability rules affect care levels of human drivers and AV manufacturers. Both studies do not assume that the AV is itself a strategic “actor”. Our analysis is distinct from these other efforts in at least two respects. First, we posit a legal rule for inferring causation, which in turn is fed into a general class of comparative negligence rules. And second, we do model the AV as having the capacity to take precautions, where its technology is the product of investments made by the manufacturer.

1.2. Literature Review

Tort law constitutes one of the oldest “common law” institutions for regulating behavior, deriving its authority from longstanding precedent that has for centuries imposed on actors a legal duty to exercise appropriate precautions when engaging in activities that are potentially harm-creating. Should a harm occur in such a context that is caused by the actor’s fault (or breach of that duty), she can be held liable in damages to foreseeable victims who suffer from damages (Anderson et al., 2014). In cases where potential victims were also active participants capable of taking precautions, a subsidiary doctrine of “contributory” or “comparative” negligence also plays a role, either eliminating or limiting (respectively) recoverable damages even when an injuring party is at fault when the injured party was negligent in taking precautions. Jurisdictions tend to vary on what degree of fault is sufficient to trigger liability, ranging from strict liability on one end of the spectrum to no-fault on the other, with various versions of “negligence” occupying the middle ground.
(Friedman and Talley, 2019). During its centuries-long existence, accident law has been forced to evolve many times, often on the heels of significant technological shocks to human interaction that involve significant risks. In the transportation sector, the advent of the automobile proved particularly significant, in large part because the automobile introduced the salient challenge of how to apportion responsibility between a human driver causing harm (who may or may not have exercised care) and the vehicle itself (manufactured by a third party). Approximately a century ago, courts began routinely to allow injured parties to bring lawsuits not only against the injuring driver(s), but also against the manufacturer of either / both vehicles, alleging that they were produced in a fashion that caused them to be dangerously unsafe (Cardozo, 1996; Hylton, 2013). Although it is often said to be a form of “strict liability,” products liability law itself also tends to vary in the extent of fault required, both to trigger potential liability and to measure comparative / contributory negligence (Hevelke and Nida-Rümelin, 2014). A now well-known literature in the economic analysis of law has studied how the various combinations of tort law and products liability are likely to perform in the context of human driven interactions (Shavell, 2007, 2009). An immediate implication of this literature is that driver interactions often involve joint investments in risk reduction – a context that is challenging to regulate efficiently, since drivers bear direct costs of their own care taking but may not enjoy the full economic benefits.

Development of liability rules requires one to first model the interactions between AVs and other road users. There has been a relatively long history of using game-theoretic approaches to model the interactions between road users and predict the behavior of these road users, when changes in road environment arise from new traffic management countermeasures (Yang and Woo, 2000; Lim et al., 2005) or technologies (Assum et al., 1999; Risa, 1992). Interested readers can refer to Elvik (2013) for a comprehensive review of these studies.

A majority of existing studies have focused on designing AVs’ driving algorithms in various scenarios to ensure traffic efficiency, safety, or ethics (Reece and Shafer, 1993; Yoo and Langari, 2013; Kim and Langari, 2014; Yu et al., 2018; Wang et al., 2015; Talebpour et al., 2015; Wang et al., 2016; Gong et al., 2016; Gong and Du, 2018; Li et al., 2018; Huang et al., 2019a,c,b; Smith, 2019), but ignoring human drivers’ behavioral adaptation to AVs as humans are exposed to more and more traffic encounters with AVs. Human drivers may have a weaker incentive to exercise “due care” when faced with AVs. Since human drivers perceive AVs as intelligent agents with the ability to adapt to more aggressive and potentially dangerous human driving behavior, the so-called “moral hazard” effect may lower human driver’s caution. It is a well-studied phenomenon in economics (Pedersen, 2001, 2003) and is also observed in traffic contexts (Boyer and Dionne, 1987; Risa, 1992; Chatterjee and Davis, 2013; Chatterjee, 2016; Millard-Ball, 2016). In a rear-end crash scenario, Chatterjee and Davis (2013) demonstrated that human drivers tended to have long reaction times and short car-following headways as the proportion of AVs on the road increases, thereby offsetting, to a certain degree, the expected benefit of AVs in reducing the risk of rear-end crashes. In another study, Millard-Ball (2016) showed that a pedestrian may cross an intersection recklessly, even if it is the AV’s right-of-way, because of the general perception that AVs, equipped with smart sensing technologies, should be able to take evasive actions in a timely manner.

1.3. Contributions of This Paper

The most relevant paper to our work is Friedman and Talley (2019), which explored how accident law should adapt to the emergence of AVs using the multilateral precaution framework from the economics of tort law. Accordingly, no fault, strict liability, and a family of negligence-based rules, i.e., fault-based products liability, are all candidates for efficient legal rules. However, it did not explore the full range of strategic interactions between the marginal cost of manufacturer precautions and the possibility that the AV itself can take precautions on the road.

This paper aims to develop a game theory model to capture the strategic interactions among different agents, namely road users comprised of AVs and HVs, the AV manufacturer, and the law maker, that coexist in the transportation ecosystem. To eliminate human drivers’ moral hazard and ensure an optimal design of autonomous driving algorithms, design of liability rules will be discussed. The contributions of this paper are:
1. Building on a good understanding of both AVs and HVs equilibrium behaviors in the developed game, we would like to explore human drivers’ moral hazard incurred by the presence of AVs.

2. We aim to model how the AV manufacturer selects safety specifications for AVs as the market becomes larger. Accordingly, the role of the AV manufacturer on traffic safety is explored.

3. In addition to modeling traffic rules, our proposed traffic flow model will allow us to simulate a variety of permutations of conventional tort law and products liability law, varying the degree of fault required to trigger a liability obligation / defense. Accordingly, a sequence of sensitivity analysis is performed to investigate how the transportation system performance may change when the related parameters vary.

The remainder of this paper is organized as follows. In Section 2, we first introduce the necessary knowledge needed to formulate games. In Section 3, a hierarchical game is formulated considering the game between road users (HVs and AVs), one between the AV manufacturer and HVs, and one between the law maker and other users. In Section 4, the mathematical properties of the game are explored, including the existence and uniqueness of the equilibrium, and an algorithm to solve the equilibrium is devised. The game and the algorithm are tested on numerical results in Section 5. Finally, conclusions and future work are presented in Section 6.

2. Preliminaries

In this paper, we use the following notations:

Notations

**Lower Level**

$c_A$ care level of autonomous vehicles, which is predetermined by the AV manufacturer’s sensors.

$c_{A_i}^{(AA)}$ care level of autonomous vehicles in the $AA$ scenario where $i = 1, 2$ and $c_{A_i}^{AA} = c_A$.

$c_A^{(AH)}$ care level of autonomous vehicles in the $AH$ scenario where $c_A^{AH} = c_A$.

$c_H^{(AH)}$ care level of human drivers in the $AH$ scenario.

$c_{H_i}^{(HH)}$ care level of human drivers in the $HH$ scenario where $i = 1, 2$.

$ub_{c_A}$ upper bound of AVs' care level.

$ub_{c_H}$ upper bound of HVs' care level.

$C_{c_A}$ feasible set for AVs' care level.

$C_{c_H}$ feasible set for HVs' care level.

$s_A^{(AH)}$ share function of an AV when a crash happens in the $AH$ scenario.

$s_H^{(AH)}$ share function of a HV when a crash happens in the $AH$ scenario.

$s_{H_i}^{(HH)}$ share function of HV player $H_i$ when a crash happens in the $HH$ scenario where $i = 1, 2$.

$s_{A_i}^{(AA)}$ share function of HV player $A_i$ when a crash happens in the $AA$ scenario where $i = 1, 2$.

$S_H$ cost of executing a care level for a human driver, also called “precaution cost”.

$C^{(AH)}_H$ cost of HVs in the AH scenario.

$C^{(HH)}_{H_i}$ cost of HVs in the HH scenario where $i = 1, 2$.

$S_A$ cost of executing a care level for an AV, also called “precaution cost” or “sensor cost”, representing the cost of sensor production for the AV manufacturer.

$C_A$ total cost for the AV manufacturer including sensor’s cost and crash loss caused by AVs.

$\alpha$ parameter of sensor cost function $S$.

$\beta$ parameter of care level cost function $u$.

$w_a$ trade-off coefficient in the AV manufacturer’s cost.

$w_h$ trade-off coefficient in human drivers’ cost.

**Upper Level**

$\sigma_H$ care level standard of HVs, measuring the relative importance of HVs’ contributions to car crashes.

$\sigma_A$ care level standard of AVs, measuring the relative importance of AVs’ contributions to car crashes.

$k$ care level standard ratio, representing the law maker’s punishments on road users where $k = \frac{\alpha_H}{\sigma_A}$.

$ub_k$ upper bound of the law maker’s decision $k$.

$C_k$ feasible set for the law maker’s decision $k$.

$SC$ social cost for a law maker, representing the sum of total crash loss and cost of precautions.

$TC$ total cost of road users’ care levels.

$w_l$ trade-off coefficient in the law maker’s payoff.

**Other Notations**

$p$ penetration rate (market share) of AVs, representing the proportion of AVs in the market.

$P$ crash probability of a car accident which is determined by road users’ care level.

$T$ average crash loss of a car accident in car scenarios, which is assumed to be a constant.

$R$ crash rate for each car scenario.

$TL$ total crash loss on the road which is the sum of crash loss in all car scenarios.

$TR$ total crash rate in the market.

$a$ parameter of crash probability function $P$, measuring contribution of HVs to crash loss

$h$ parameter of crash probability function $P$, measuring contribution of AVs to crash loss
2.1. Crash for Three Types of Vehicle Encounters

On the road, there are three types of vehicular encounters. They are:

1. an HV encounters the other HV (i.e., $HH$ scenario). The two players in this scenario are denoted as $H_1^{HH}$ and $H_2^{HH}$, respectively;

2. an AV encounters an HV (i.e., $AH$ scenario). The two players in this scenario are denoted as $A^{(AH)}$ and $H^{(AH)}$, respectively;

3. an AV encounters the other AV (i.e., $AA$ scenario). The two players in this scenario are denoted as $A_1^{(AA)}$ and $A_2^{(AA)}$, respectively.

We first assume a designated market penetration rate of AVs and the probability of three types of vehicle encounters can be computed. Define $p$ as the AVs’ market penetration rate. Accordingly, the probabilities of three encounters are $(1 - p)^2$ for the $HH$ scenario, $2p(1 - p)$ for the $AH$ scenario, and $p^2$ for the $AA$ scenario, respectively. The sum of three probabilities equals one.

Based on different values of $p$, three markets can be defined, shown in Table 1. The market is a pure HV market when $p = 0$. In a pure HV market, there is only one type of vehicle encounter, which is the $HH$ scenario. The market is a pure AV market when $p = 1$. In a pure AV market, there is only one type of vehicle encounter, which is the $AA$ scenario. The market is a mixed AV-HV market when $0 < p < 1$. In a mixed AV-HV market, there are three types of vehicle encounters, which are the $HH$, $AA$, and $AA$ scenarios.

<table>
<thead>
<tr>
<th>Vehicle Encounter</th>
<th>Market</th>
<th>Pure HV Market ($p = 0$)</th>
<th>Mixed AV-HV Market ($0 &lt; p &lt; 1$)</th>
<th>Pure AV Market ($p = 1$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>HV-HV</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>AV-HV</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>AV-AV</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td></td>
</tr>
</tbody>
</table>

Table 1: Three markets with vehicular encounter scenarios

Within one encounter, an accident may or may not happen. The occurrence of an accident depends on the care level of two vehicles. Care level is an abstract quantity to represent how attentive one driver is while driving: A higher care level implies a more cautious attitude to road safety. For human drivers, it can be specified as the difference between reaction time and following headway in the context of rear-end crashes (Chatterjee and Davis, 2013; Chatterjee, 2016). For AVs, their care level, determined by the AV manufacturer in the production process, represents a safety specification level (Shavell, 2019). Such a specification is determined by a combined performance of sensing, computational, and actuating units.

Sensors, including video camera, ultrasonic sensors, and Lidar, are by far one of the biggest obstacles in vehicle autonomy. Accordingly, in this paper, we will use sensor quality to represent AVs’ safety level. The more expensive sensors are deployed, the higher care level AVs take, and the safer these AVs are. We use $c_i^{(HH)}, i = 1, 2$ to represent HVs’ care level in the $HH$ scenario, $c_i^{(AA)}, i = 1, 2$ to represent AVs’ care level in the $AA$ scenario and $c_i^{(AH)}, i = 1, 2$ to represent HVs’ and AVs’ care level in the $AH$ scenario.

The higher care level one vehicle chooses, the less likely an accident happens. We thus define a crash probability function between vehicles $i$ and $j$ as $P(c_i, c_j)$, which is monotonically decreasing with respect to care levels. In other words, $\frac{\partial P(c_i, c_j)}{\partial c_i} < 0$, $\frac{\partial P(c_i, c_j)}{\partial c_j} < 0$ (Pedersen, 2003). Crash probability functions for each scenario are $P(c_1^{(HH)}, c_2^{(HH)})$, $P(c_1^{(AH)}, c_2^{(AH)})$ and $P(c_1^{(AA)}, c_2^{(AA)})$. In addition, we assume average crash loss of all scenarios is the same, denoted by a constant $T$. Mathematically, crash loss of each scenario is denoted by $P(c_1^{(AA)}, c_2^{(AA)}) \cdot T$, $P(c_1^{(HH)}, c_2^{(HH)}) \cdot T$ and $P(c_1^{(AH)}, c_2^{(AH)}) \cdot T$.

Based on encounter probability and crash probability, we define crash rate for each scenario. Crash rate shows the proportion of cars involved in crashes, equalling the product of encounter probability and crash...
probability. Accordingly, the crash rates are
\( R(HH), R(AA), R(AH) \) for the HH,AA, AH scenarios, respectively. The share function for the HH scenario is
\( (c_{HH}^{(i)}, c_{HH}^{(j)}) = (1-p)^2 \cdot P(c_{HH}^{(i)}, c_{HH}^{(j)}) \) for the HH scenario,
\( R(AA), R(AH) \) for the AA and AH scenarios, respectively. Total crash rate equals the sum of crash rates of all scenarios:
\[ TR(c_{HH}^{(i)}, c_{HH}^{(j)}) = R(c_{HH}^{(i)}, c_{HH}^{(j)}) + R(c_{AA}^{(i)}, c_{AA}^{(j)}) + R(c_{AH}^{(i)}, c_{AH}^{(j)}). \] Note that although AVs are likely to have greater powers in perception with a higher standard of care than human drivers, car crashes may still occur in the AA scenario (Shavell, 2019).

2.2. Share Function

A liability rule is a mechanism to apportion direct costs associated with an accident between involved drivers. As we mentioned in the previous section, Friedman and Talley (2019); Shavell (2019) discussed the possibility of using no fault, strict liability, and negligence-based liability policies for AVs. In this paper, we will use negligence-based liability, comparative negligence in particular, for both driver liability and product liability to regulate human’s driving behavior and AVs’ driving algorithms.

Negligence-based liability, including contributory and comparative negligence (T. Schwartz, 1978; Parisi and Fon, 2004; P. B. De Mot, 2012), are primarily designed to incentivize one to exercise “due care”, which is a predefined care level threshold. Contributory negligence stipulates that the negligent driver pays for the entire crash loss. If both drivers are negligent, they have the same liability. Comparative negligence, on the other hand, divides crash loss according to drivers’ relative contribution to a crash. In other words, one’s negligence can be quantified by the difference between her care level and a predefined standard, which is a desirable care level one should execute while driving on the road. A causation function – “share function” is used to measure one’s loss share based on her comparative negligence level to the care level standard. Different causation functions have been proposed in the literature (Parisi and Fon, 2004; Ram, 2007; Chatterjee, 2016; Friedman and Talley, 2019). For example, one's causation contribution depended on both care level and activity level in Parisi and Fon (2004); Ram (2007). Assuming a constant driving activity level, Chatterjee (2016); Friedman and Talley (2019) defined one’s causation contribution as the ratio of her relative care level to the care level standard, respectively. The share function is then reformulated as:
\[ s_i(c_i, c_j, \sigma_i, \sigma_j) = \frac{e^{e_i^j - c_j}}{e^{e_i^j - c_i} + e^{e_j^i - c_j}}, \] where \( s_i \) is the share function of road user \( i \). \( e_i^j, c_j \) are care level of road users \( i \) and \( j \) in a crash, respectively. \( \sigma_i, \sigma_j \) are care level standards for road users \( i \) and \( j \), respectively. To simplify it, we convert the exponential form into a linear fractional form by substituting \( e_i^j \) with \( c_i, e_j^i \) with \( c_j, e_i^j \) with \( \sigma_i, e_j^i \) with \( \sigma_j \), where \( c_i, c_j \) are care level of road user \( i \) and \( j \) in a crash, and \( \sigma_i, \sigma_j \) are care level standards for road user \( i \) and \( j \), respectively. The share function is then reformulated as:
\[ s_i(c_i, c_j, \sigma_i, \sigma_j) = \frac{\sigma_i c_i^{-1}}{\sigma_i c_i^{-1} + \sigma_j c_j^{-1}}, \]

Define the care level standards for HVs and AVs as \( \sigma_H \) and \( \sigma_A \), respectively. The share function for the HH scenario is:
\[ s_{HH}(c_{HH}, c_{HH}, \sigma_H) = \frac{\sigma_H (c_{HH})^{-1}}{\sigma_H (c_{HH})^{-1} + \sigma_H (c_{HH})^{-1}}, \]
where \( s^{(HH)}_H \) is the share function and \( c^{(HH)}_H \) is the care level of HV player \( H_i \).

The share function for the \( AH \) scenario is:

\[
s^{(AH)}_A (c^{(AH)}_A, c^{(AH)}_H, \sigma_A, \sigma_H) = \frac{\sigma_A (c^{(AH)}_A)^{-1}}{\sigma_H (c^{(AH)}_H)^{-1} + \sigma_A (c^{(AH)}_A)^{-1}},
\]

\[
s^{(AH)}_H (c^{(AH)}_A, c^{(AH)}_H, \sigma_A, \sigma_H) = \frac{\sigma_H (c^{(AH)}_H)^{-1}}{\sigma_H (c^{(AH)}_H)^{-1} + \sigma_A (c^{(AH)}_A)^{-1}},
\]

where \( s^{(AH)}_A, s^{(AH)}_H \) are share functions and \( c^{(AH)}_A, c^{(AH)}_H \) are care levels of AV player \( A \) and HV player \( H \).

The share function for the \( AA \) scenario is:

\[
s^{(AA)}_{A_i} (c^{(AA)}_{A_1}, c^{(AA)}_{A_2}, \sigma_A) = \frac{\sigma_A (c^{(AA)}_{A_i})^{-1}}{\sigma_A (c^{(AA)}_{A_1})^{-1} + \sigma_A (c^{(AA)}_{A_2})^{-1}}, \quad i = 1, 2,
\]

where \( s^{(AA)}_{A_i} \) is the share function and \( c^{(AA)}_{A_i} \) is the care level of AV player \( A_i \).

### 2.3. Crash Loss Share

A share function apportions an expected crash loss of an accident between two involved parties. Denote the average crash loss of a car accident as \( T \). Then the expected crash loss for an accident is the product of crash probability and average loss. Accordingly, one’s crash loss share is an expected crash loss of an accident times its share.

In the \( HH \) scenario, the crash loss share for player \( H_i^{(HH)} \) is:

\[
P(c^{(HH)}_{H_1}, c^{(HH)}_{H_2}) \cdot s^{(HH)}_{H_1} (c^{(HH)}_{H_1}, c^{(HH)}_{H_2}, \sigma_{H_1}, \sigma_{H_2})
\]

where \( i = 1, 2 \).

In the \( AH \) scenario, the crash loss share for player \( H^{(AH)} \) is:

\[
P(c^{(AH)}_A, c^{(AH)}_H) \cdot s^{(AH)}_H (c^{(AH)}_H, c^{(AH)}_A, \sigma_A, \sigma_H)
\]

and that for player \( A^{(AH)} \) is:

\[
P(c^{(AH)}_A, c^{(AH)}_H) \cdot s^{(AH)}_A (c^{(AH)}_A, c^{(AH)}_H, \sigma_A, \sigma_H).
\]

In the \( AA \) scenario, the crash loss share for player \( A^{(AA)}_i \) is:

\[
P(c^{(AA)}_{A_1}, c^{(AA)}_{A_2}) \cdot s^{(AA)}_{A_i} (c^{(AA)}_{A_1}, c^{(AA)}_{A_2}, \sigma_A)
\]

where \( i = 1, 2 \).

To summarize, Table 2 shows crash probability, crash rate, and crash loss share for each vehicle encounter scenario.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Player</th>
<th>Crash probability</th>
<th>Crash rate</th>
<th>Crash loss share</th>
</tr>
</thead>
<tbody>
<tr>
<td>HH</td>
<td>( H^{(HH)}_1, H^{(HH)}_2 )</td>
<td>( p(c^{(HH)}<em>{H_1}, c^{(HH)}</em>{H_2}) ) ( (1 - p)^2 p(c^{(HH)}<em>{H_1}, c^{(HH)}</em>{H_2}) ) ( P(c^{(HH)}<em>{H_1}, c^{(HH)}</em>{H_2}) \cdot s^{(HH)}_{H_1} ) ( i = 1, 2 )</td>
<td>( p(c^{(HH)}<em>{H_1}, c^{(HH)}</em>{H_2}) ) ( (1 - p)^2 p(c^{(HH)}<em>{H_1}, c^{(HH)}</em>{H_2}) ) ( P(c^{(HH)}<em>{H_1}, c^{(HH)}</em>{H_2}) \cdot s^{(HH)}_{H_1} ) ( i = 1, 2 )</td>
<td>( p(c^{(HH)}<em>{H_1}, c^{(HH)}</em>{H_2}) ) ( (1 - p)^2 p(c^{(HH)}<em>{H_1}, c^{(HH)}</em>{H_2}) ) ( P(c^{(HH)}<em>{H_1}, c^{(HH)}</em>{H_2}) \cdot s^{(HH)}_{H_1} ) ( i = 1, 2 )</td>
</tr>
<tr>
<td>AH</td>
<td>( A^{(AH)}_1, A^{(AH)}_2 )</td>
<td>( p(c^{(AH)}_{A_1}, c^{(AH)}<em>H) ) ( 2(1 - p)p(c^{(AH)}</em>{A_1}, c^{(AH)}<em>H) ) ( P(c^{(AH)}</em>{A_1}, c^{(AH)}_H) \cdot s^{(AH)}_H ) ( i = 1, 2 )</td>
<td>( p(c^{(AH)}_{A_1}, c^{(AH)}<em>H) ) ( 2(1 - p)p(c^{(AH)}</em>{A_1}, c^{(AH)}<em>H) ) ( P(c^{(AH)}</em>{A_1}, c^{(AH)}_H) \cdot s^{(AH)}_H ) ( i = 1, 2 )</td>
<td>( p(c^{(AH)}_{A_1}, c^{(AH)}<em>H) ) ( 2(1 - p)p(c^{(AH)}</em>{A_1}, c^{(AH)}<em>H) ) ( P(c^{(AH)}</em>{A_1}, c^{(AH)}_H) \cdot s^{(AH)}_H ) ( i = 1, 2 )</td>
</tr>
<tr>
<td>AA</td>
<td>( A^{(AA)}_1, A^{(AA)}_2 )</td>
<td>( p(c^{(AA)}<em>{A_1}, c^{(AA)}</em>{A_2}) ) ( p^2 p(c^{(AA)}<em>{A_1}, c^{(AA)}</em>{A_2}) ) ( P(c^{(AA)}<em>{A_1}, c^{(AA)}</em>{A_2}) \cdot s^{(AA)}_{A_1} ) ( i = 1, 2 )</td>
<td>( p(c^{(AA)}<em>{A_1}, c^{(AA)}</em>{A_2}) ) ( p^2 p(c^{(AA)}<em>{A_1}, c^{(AA)}</em>{A_2}) ) ( P(c^{(AA)}<em>{A_1}, c^{(AA)}</em>{A_2}) \cdot s^{(AA)}_{A_1} ) ( i = 1, 2 )</td>
<td>( p(c^{(AA)}<em>{A_1}, c^{(AA)}</em>{A_2}) ) ( p^2 p(c^{(AA)}<em>{A_1}, c^{(AA)}</em>{A_2}) ) ( P(c^{(AA)}<em>{A_1}, c^{(AA)}</em>{A_2}) \cdot s^{(AA)}_{A_1} ) ( i = 1, 2 )</td>
</tr>
</tbody>
</table>

Table 2: Quantities for three vehicular encounter scenarios

### 3. Game Formulations

In this section, we will state the problem and then introduce its mathematical formulations. Assumptions we will use for the game-theoretic framework are first presented:

1. There is only one AV manufacturer that produces one type of sensor. Thus all AVs on the road use the same type of sensors. Mathematically, \( c^{(AH)}_A = c^{(AA)}_A = c_A \equiv c_A \). Accordingly, when there is an accident happening between two AVs, the AV manufacturer endures the entire crash loss.

2. The more expensive the sensors are, the less likely a crash happens; and vice versa. Define the sensor cost as \( S_A(c_A) \), which is an increasing function with respect to \( c_A \). Mathematically, \( \frac{dS_A(c_A)}{dc_A} > 0 \).
3. To achieve the same care level $c$, the cost of precautions for AVs (i.e., sensor cost) should be lower than that for HVs. Mathematically, $\frac{dS_H(c_H)}{dc_H}|_{c_H=c} > \frac{dS_A(c_A)}{dc_A}|_{c_A=c} > 0$, where $S_H(c_H)$ is the precaution cost for HVs to exercise care level $c_H$, an increasing function with respect to $c_H$. In addition, AVs can achieve higher care level than HVs, mathematically, $ub_{c_A} \geq ub_{c_H}$.

4. Human drivers share the same care level standard $\sigma_H$. Also, all AVs share the same care level standard $\sigma_A$. Accordingly, the share function in the $HH$ scenario can be simplified to $s_{H_i}^{(HH)} = \frac{(c_{H_1}^{(HH)})^{-1} \cdots (c_{H_i}^{(HH)})^{-1}}{(c_{H_1}^{(HH)})^{-1} + \cdots + (c_{H_2}^{(HH)})^{-1}}$, $i = 1, 2$. The share function in the $AA$ scenario can be reduced to $s_{A_1}^{(AA)} = s_{A_2}^{(AA)} = \frac{1}{2}$. Because these two reduced share functions do not contain care level standard any more, it says that the law maker plays no role in risk apportion in neither the pure HV nor the pure AV markets.

5. Three vehicular encounter scenarios are independent of one another. In other words, one’s care level change in one scenario does not influence her choice in other scenarios. For example, human drivers only adapt their care levels in the $AH$ scenario while encountering AVs but maintain the same care levels in the $HH$ scenario while encountering other human drivers.

6. Human drivers have complete information about vehicle encounter scenarios. In other words, they know whether it is an AV or another HV they encounter.

A mixed traffic system is comprised of hierarchical decision makers who make choices at different levels:

1. Law-maker: the leader (i.e., the top level player) develops an optimal driver liability rule for human drivers and a product liability rule for AV manufacturers, aiming to minimize the social cost (i.e., the total crash loss plus the total cost of care levels);

2. AV manufacturer: the mid-level player determines driving settings or AVs’ care level in order to minimize its expected payoff over all possible traffic crashes involved with AVs (i.e., the sensor cost plus the expected crash loss). In this paper, we assume there is only one AV manufacturer. Its generalization to multiple AV manufacturers will be left for future research.

The AV manufacturer makes a one-time investment regarding what types of sensors to purchase. Once sensors are chosen and installed, AVs are put on public roads and may subject to potential risks of crashing into other vehicles.

3. Human driver: the lower-level player (i.e., the follower) selects the level of care to maximize her own utility, which is her cost of executing precaution plus the crash loss for one accident.

One driver can be involved in an accident with another HV or with an AV. For the accident between two HVs, the driver liability rule is applied to apportion the crash loss. For the accident between one HV and one AV, the driver liability rule is applied to the HV while the product liability rule is applied to the AV.

In summary, on the top level, the law maker is the leader to make a high-level decision on the driver liability rule and the product liability rule. Once the legal framework is formed, AV manufacturers decide on the strategic level as to what safety levels their AVs are by selecting sensors that determine one AV’s level of care while driving on roads. On public roads, human drivers make an operational decision regarding the level of care while driving. To model decisions of various decision-makers on different levels, a hierarchical game-theoretic framework, leader-follower game in particular, needs to be developed. Figure 1 illustrates the structure of the game framework for three markets, respectively.
In the next three subsections, we will introduce the game on each level, from the bottom level to the top level. In each game, we will define players’ decision variable and disutility functions, the game formulation, and the equilibrium condition. The games will be parameterized by the AV penetration rate \( p \), to reflect the impact of AV fleet size on each game.

3.1. Game Between Human Drivers

On the lower level game, human drivers are strategic game players interacting with one another on public roads to minimize their own cost by choosing care levels. A two-player simultaneous Nash game is developed for the \( HH \) scenario that could happen in both the pure HV market and the mixed AV-HV market. The set-up of this game is similar to the road safety game proposed by Pedersen (2001).

**Players:** Human drivers \( H_i (i = 1, 2) \) play a symmetric pairwise Nash game.

**Decision Variables:** The driving decision the human driver \( H_i \) chooses in a road game is her care level or the level of precaution in the \( HH \) scenario, denoted as \( c_{HH} \). Define \( C_{HH} : (0, ub_{HH}) \subset R^+ \) as the feasible set for players’ care-level and \( ub_{HH} \) is the upper bound.

**Cost Function:** While driving, human drivers aim to maintain traffic safety (i.e., to minimize total crash loss) and efficiency (i.e., to execute less care level if possible) (Pedersen, 2001; Chatterjee and Davis, 2013). The cost function of a human driver \( H_i \) in the \( HH \) scenario, denoted as \( C_{HH} (c_{HH}, c_{HH}) \), is comprised of two parts: the cost of executing a care level and the crash loss share. Denote \( S_H (c_{HH}) \) as the cost of executing a care level for driver \( H_i \): the higher the care level is executed, the higher the cost is, i.e., \( \frac{dS_H (c_{HH})}{dc_{HH}} > 0 \). Considering the crash loss share defined in Section 2.3, then the disutility function can be
formulated as:

$$C_{H_i}^{(HH)}(c_{H_1}^{(HH)}, c_{H_2}^{(HH)}) = w_h \cdot S_H(c_{H_i}^{(HH)}) + P(c_{H_1}^{(HH)}, c_{H_2}^{(HH)}) \cdot T \cdot S_H(c_{H_1}^{(HH)}, c_{H_2}^{(HH)}), i = 1, 2.$$ \hspace{1cm} (3.1)$$

where $w_h$ is a trade-off coefficient in human drivers’ cost function.

**Game:** Each human driver makes a trade-off between minimization of care level and minimization of crash loss.

$$\text{[GameHH]} \min_{c_{H_i}^{(HH)} \in C_{H_i}} C_{H_i}^{(HH)}(c_{H_1}^{(HH)}, c_{H_2}^{(HH)}), i = 1, 2$$ \hspace{1cm} (3.2)

The equilibrium of this game, denoted as $(c_{H_1}^{*(HH)}, c_{H_2}^{*(HH)})$, represents the optimal care level for human drivers $H_1, H_2$.

**Remark.** According to Assumption 5, the equilibrium care level solved from [GameHH] for the HH scenario remains the same in both the pure HV market and the mixed AV-HV market. In other words, an HV selects the same care level whenever she meets another HV, regardless of the market.

**Equilibrium condition:** At equilibrium, no human drivers can improve her utility by unilaterally switching her care level, i.e., $C_{H_i}^{(HH)}(c_{H_i}^{*(HH)}) < C_{H_i}^{(HH)}(c_{H_i}^{(HH)}), \forall c_{H_i}^{(HH)} \in C_{H_i}$.

### 3.2. Game Between the AV Manufacturer and HVs

In the AV-HV mixed market when $p \in (0, 1)$, human drivers do not only interact with one another in the $HH$ scenario, but also interact with AVs in the $AH$ scenario. Unlike human drivers, AVs are not strategic players because their care levels are predetermined by the AV Manufacturer. The AV Manufacturer trades off between investment cost of sensors and an average crash loss over all possible accidents involved with AVs. Accordingly, the AV Manufacturer and HVs play a leader-follower game in which the AV Manufacturer stipulates a care level for its AV fleet and human drivers select their care levels while encountering AVs on roads.

**Players:** The AV Manufacturer is a leader and HVs are followers.

**Decision Variables:** The AV manufacturer makes a one-time investment on what types of sensors to purchase, which is a trade-off between sensor investment and the total crash loss involved with AVs. Denote the care level of AVs as $c_A$ and the precaution cost associated with the care level $c_A$ as $S_A(c_A)$, named “sensor cost” for short. Because AVs’ care level is critically determined by sensors, the cost associated with care level will be also referred to as sensor (production) cost in the rest of the paper. Human drivers’ decision variables are care levels $c_A^{(AH)}$ as defined before. In addition, feasible sets for $c_A, c_A^{(AH)}$ are $C_{c_A} : (0, ub_{c_A}) \subset R^+$ and $C_{c_A^{(AH)}} : (0, ub_{c_A^{(AH)}}) \subset R^+$, respectively.

**Cost Function:** The AV manufacturer’s cost function, denoted as $C_A(c_A, c_A^{(AH)})$, is the sensor cost plus the expected crash loss. To calculate the crash loss, two scenarios are relevant to the AV manufacturer: $AH$ and $AA$ encounters. The cost is calculated as:

$$C_A(c_A, c_A^{(AH)}) = w_a \cdot S_A(c_A) + \begin{pmatrix} \text{Sensor cost} \\ \text{AVs’ loss share in the AA scenario} \\ \text{AV’s loss share in the AH scenario} \end{pmatrix} (p^2 \cdot P(c_A, c_A^{(AH)}) \cdot T) + 2p(1-p) \cdot P(c_A, c_A^{(AH)}) \cdot T \cdot S_A^{(AH)}(c_A, c_A^{(AH)}).$$ \hspace{1cm} (3.3)

where $w_a$ is a trade-off coefficient in the AV manufacturer’s cost function.

On public roads, non-strategic AV players and strategic HV players interact with one another. The care level for non-strategic AVs, i.e., $c_A$, is predetermined through minimization of the AV manufacturer’s cost function. Strategic HV players choose care levels to minimize their own cost when they encounter AVs. Similar to Equation 3.1, human drivers’ disutility in the $AH$ scenario, denoted as $C_{H}^{*(AH)}(c_A^{(AH)}, c_A)$, is
defined as

\[
C_{AH}(c_A^{(AH)}, c_H^{(AH)}) = w_h \cdot S_H(c_H^{(AH)}) + P(c_A, c_H^{(AH)}) \cdot T \cdot s_H^{(AH)}(c_A, c_H^{(AH)}). \tag{3.4}
\]

**Game:** The AV manufacturer aims to select a set of optimal sensors to minimize its total cost, which depends on the care level of HV players in the \(AH\) scenario. Accordingly, the AV manufacturer and human drivers are the leader and the follower, respectively. Given \(p \in (0, 1)\), the game is formulated as:

\[
\text{[GameAH]} \min_{c_A \in C_{AH}} C_A(c_A, c_H^{(AH)}) \\
\text{s.t.} \quad c_H^{(AH)} \in \arg \min_{c_H^{(AH)} \in C_{AH}} C_H^{(AH)}(c_H^{(AH)}, c_A)
\tag{3.5}
\]

The equilibrium of the game is denoted by \((c_A^*(p), c_H^{(AH)}(p))\), \(p \in (0, 1)\).

**Equilibrium condition:** At equilibrium, the AV manufacturer and human drivers cannot lower their cost by unilaterally switching their respective decisions, i.e., \(C_A(c_A(p), c_H^{(AH)}(p)) < C_A(c_A, c_H^{(AH)}(p)), \forall c_A \in C_{AH}\) and \(C_H^{(AH)}(c_H^{(AH)}(p), c_A) < C_H^{(AH)}(c_H^{AH}, c_A), \forall c_H^{(AH)} \in C_{c_H}\).

In the pure AV market, there does not exist any HVs. According to Assumption 1, we assume there is only one AV manufacturer, so the pure AV market is essentially a monopoly market. Instead of solving the above game, the AV manufacturer solves an optimization problem for sensor investment. Given \(p = 1\),

\[
\text{[GameAV]} \min_{c_A \in C_A} C_A(c_A) = w_a \cdot S_A(c_A) + P(c_A, c_A) \cdot T
\tag{3.6}
\]

The optimal solution is denoted by \(c_A^*(p), p = 1\).

### 3.3. Game Between the Law Maker and Others

Individual’s optimum may deviate from social welfare in most cases because of selfishness. Each player involved in the hierarchical game has different and possibly opposing objectives. This conflict may be accentuated when AVs are preprogrammed with algorithmic calculus and face dilemmas in decision-making process when social ethics are involved. The trade-off between social welfare and individual benefits is thus crucial in the AV-HV context.

On the top level, the law maker develops a driver liability rule for human drivers and a product liability rule for the AV manufacturer, to minimize the social cost. Based on the liability policies, an optimal driving strategy has to be in-built within AVs’ design guidelines. We would like to understand the impact of various liability policies on the decision-making of the AV manufacturer and road users and identify a set of liability policies under which an optimal balance between efficiency and safety could be achieved for AVs. For example, an overtly “risk-averse” AV may potentially impact its own and other AVs driving efficiency, and create moral hazard effect on human drivers (Millard-Ball, 2016). On the other hand, an aggressively designed AV (with lower weights on safety costs) has a higher risk of suffering penalty in the occurrence of an accident, thanks to product liability. Thus the future liability policies will heavily influence how AV manufacturers select cost coefficients and how human drivers react to AVs.

**Players:** The law maker is a leader, while the AV Manufacturer and HVs are followers.

**Decision Variables:** In a liability rule, the law maker determines the care level standards \(\sigma_H > 0\) and \(\sigma_A > 0\) appearing in share functions. According to Assumption 4, the law maker does not play a role in neither the pure HV nor the pure AV market. Therefore the game developed on this level is only concerning the mixed HV-AV market. Define the strategy of the law maker as the care level standard ratio, denoted as \(k \equiv \frac{\sigma_H}{\sigma_A} (k > 0)\). The care level standard ratio \(k\) can be interpreted as a punishment ratio for different road users in liability rules. \(k > 1\) implies that the standard care level of a human driver is higher than that of an AV. In other words, the law maker is stricter with HVs than AVs. \(k < 1\) implies that the law maker is stricter with AVs than HVs. \(k = 1\) implies that AVs and HVs have the same care level standard and the law
maker treats them equally. Define \( C_k : (0, u_b k) \subseteq R^+ \) as the feasible set for the law maker’s decision \( k \) and \( u_b k \) is the upper bound.

**Cost Function:** The law maker aims to find an optimal care level standard ratio for HVs’ and AVs’ liability to minimize the social cost.

Social cost is the total crash loss plus the total cost of care levels. Summarizing all three scenarios, the total crash loss \( TL \) equals the sum of crash loss across three vehicular encounters, mathematically,

\[
TL(c_{HH_1}^{(HH)}, c_{HH_2}^{(HH)}, c_A(k), c_H^{(AH)}(k)) = p^2 \cdot P(c_A(k), c_A(k)) \cdot T + 2p(1 - p) \cdot P(c_A(k), c_H^{(AH)}(k)) \cdot T
\]

\[
\text{crash loss in the AA scenario}
\]

\[
+ (1 - p)^2 \cdot P(c_{HH_1}^{(HH)}, c_{HH_2}^{(HH)}) \cdot T
\]

\[
\text{crash loss in the AH scenario}
\]

\[
+ (1 - p)^2 \cdot P(c_{HH_1}^{(HH)}, c_{HH_2}^{(HH)}) \cdot T
\]

\[
\text{crash loss in the HH scenario}
\]

Equation (7.7)

Note that \( c_A \) and \( c_H^{(AH)} \) are care levels for AVs and HVs, respectively, in the AV-HV scenario. They both are affected by the law maker’s decision \( k \) via the cost functions of HVs and AVs in the \( AH \) scenario defined in Equations 3.3 and 3.4. Therefore, we will abuse the notations from now on by using \( c_A(k) \) and \( c_H^{(AH)}(k) \) and \( C_A(c_A, c_H^{(AH)}(k)), C_H^{(AH)}(c_H^{(AH)}, c_A, k) \).

The total cost of care levels \( TC \) is computed as the sum of all road users’ care level, mathematically,

\[
TC(c_{HH_1}^{(HH)}, c_{HH_2}^{(HH)}, c_A(k), c_H^{(AH)}(k)) = p^2 \cdot (2c_A(k)) + 2p(1 - p) \cdot (c_A(k) + c_H^{(AH)}(k))\]

\[
\text{cost of care levels in the AA scenario}
\]

\[
+ (1 - p)^2 \cdot (c_{HH_1}^{(HH)} + c_{HH_2}^{(HH)})\]

\[
\text{cost of care levels in the HH scenario}
\]

Equation (7.8)

To sum up, the social cost is computed as:

\[
SC(c_{HH_1}^{(HH)}, c_{HH_2}^{(HH)}, c_A(k), c_H^{(AH)}(k)) = w_1 \cdot TC(c_{HH_1}^{(HH)}, c_{HH_2}^{(HH)}, c_A(k), c_H^{(AH)}(k)) + TL(c_{HH_1}^{(HH)}, c_{HH_2}^{(HH)}, c_A(k), c_H^{(AH)}(k))
\]

\[
\text{total cost of care levels}
\]

\[
\text{total crash loss}
\]

Equation (7.9)

where \( w_1 \) is a trade-off weight coefficient in the social cost function.

**Game:** The hierarchical game contains two levels of Stackelberg games. In the upper level, the law maker is the leader and all others are followers; while in the lower level, the AV manufacturer is the leader and human drivers are followers. Summarizing the three levels of games defined in Equations 3.2 and 3.5 gives us a hierarchical game:

\[
\begin{align*}
\text{[GameSum]} & \quad \min_{k \in C_k} SC(c_{HH_1}^{(HH)}, c_{HH_2}^{(HH)}, c_A(k), c_H^{(AH)}(k)), \quad 0 < p < 1 \\
\text{s.t. [GameHH]} & \quad c_{HH_1}^{(HH)} \in \arg\min_{c_{HH_1}^{(HH)} \in C_{HH_1}} C_{HH_1}^{(HH)}(c_{HH_1}^{(HH)}, c_{HH_2}^{(HH)}), \quad i = 1, 2 \\
\text{[GameAH]} & \quad c_A \in \arg\min_{c_A \in C_A} C_A(c_A, c_H^{(AH)}, k) \\
\text{s.t.} & \quad c_H^{(AH)} \in \arg\min_{c_H^{(AH)} \in C_H^{(AH)}} C_H^{(AH)}(c_H^{(AH)}, c_A, k)
\end{align*}
\]

Equation (3.10)

**Equilibrium condition:**

The equilibrium of the hierarchical game is denoted by \( e^* = (c_{HH_1}^{(HH)}, c_{HH_2}^{(HH)}, c_A^*(k^*, p), c_H^{(AH)}(k^*, p), k^*(p)) \)

At equilibrium, no player can improve her total cost by unilaterally switching strategies, i.e.,

\[
\begin{align*}
SC(c_{HH_1}^{(HH)}, c_{HH_2}^{(HH)}, c_A, c_H^{(AH)}(k)) \leq SC(c_{HH_1}^{(HH)}, c_{HH_2}^{(HH)}, c_A, c_H^{(AH)}(k)), \\
C_A(c_A, c_H^{(AH)}(k)) \leq C_A(c_A, c_H^{(AH)}(k)), \quad C_H^{(AH)}(c_H^{(AH)}), \quad C_A, k) \leq C_H^{(AH)}(c_H^{(AH)}(c_A, k), k), \quad C_H^{(AH)}(c_H^{(AH)}(c_A, k), k) \leq C_H^{(AH)}(c_H^{(AH)}(c_A, k), k), \quad C_H^{(AH)}(c_H^{(AH)}(c_A, k), k) \leq C_H^{(AH)}(c_H^{(AH)}(c_A, k), k), \quad i = 1, 2.
\end{align*}
\]
3.4. Performance Measures

In this subsection, we will define three performance measures to evaluate the impact of AV technologies on traffic.

Definition 3.1. 1. (Moral Hazard.) We say that a moral hazard happens to a road user if the following condition holds:
\[ c^*_i(x) > c^*_i(x') \] (3.11)

where \( c^*_i \) is road user \( i \)'s equilibrium care level. In this paper, \( x \) represents road environment or other road users’ care level and \( x' \) represents improved road environment or other road users’ care level. In other words, a moral hazard happens if a road user chooses a lower care level when others’ care level or road environment is improved.

2. (Road Safety.) We say that a pure AV market improves road safety if the following condition holds:
\[ TR^{(mixed)}(c^{(HH)}_{H1}, c^{(HH)}_{H2}, c^*_A, c^*_AH) > TR^{(pureAV)}(c^*_A), \] (3.12)

where \( TR^{(mixed)} \) and \( TR^{(pureAV)} \) are total crash rate of a mixed AV-HV and a pure AV markets, respectively.

3. (Social Welfare Improving Property.) We say that a pure AV market improves total social welfare if the following condition holds:
\[ SC^{(mixed)}(c^{(HH)}_{H1}, c^{(HH)}_{H2}, c^*_A, c^*_AH) > SC^{(pureAV)}(c^*_A), \] (3.13)

where \( SC^{(mixed)} \) and \( SC^{(pureAV)} \) are equilibrium social cost of a mixed AV-HV and a pure AV markets, respectively.

4. Analytical Properties and Algorithm

In this section, we show existence and uniqueness of the equilibrium and develop an algorithm to solve the proposed game.

4.1. Solution Existence and Uniqueness

The hierarchical game \([\text{GameSum}]\) defined in Equation 3.10 is essentially a tri-level game, with a simultaneous game in the lower level. The leader, the sub-leader, and the follower in this tri-level game correspond to the law maker, the AV manufacturer, and human drivers, respectively. The simultaneous game is the symmetric game between human drivers. To show solution existence and uniqueness, we will first introduce lemmas related to a bi-level game in the context of road safety and a simultaneous game, respectively. Then we apply these conditions to our tri-level game with the lower-level simultaneous game.

Lemma 1. (Leitmann, 1978; Pedersen, 2003) A general Stackelberg game is defined as follows:

\[
\begin{align*}
\max_{x_l} & \quad g^l(x_l, x_f) \\
\text{s.t.} & \quad x_f \in \arg \max_{x_f} g^l(x_l, x_f) \\
& \quad x_f \in X_f, \quad x_l \in X_l
\end{align*}
\] (4.1)

where \( x_l, x_f \) are decisions of a leader and a follower, \( g^l, g^f \) are their payoff functions and \( X_l, X_f \) are feasible sets for players’ decisions. The sufficient conditions for the existence and uniqueness of equilibrium \((x^*_f, x^*_l)\) are:

A The solution to \( \frac{\partial g^l(x_f, x_l)}{\partial x_l} \) = 0 is a singleton, denoted by \( x_f \equiv m_l(x_l) \).
B The solution to \( \frac{dg^i(x_i,m_i(x_i))}{dx_i} = 0 \) is a singleton, denoted by \( x^*_i \).

Furthermore, Pedersen (2003) applied the general Stackelberg game to a road safety leader-follower game and provided more specific sufficient conditions for existence and uniqueness of the equilibrium. Mathematically,

1. \( \frac{\partial^2g^i(x_i,x_j)}{\partial x_i} > 0 \) where \( g^i \) is the follower’s cost function.

2. \( \frac{d^2g^i(x_i,m_i(x_i))}{dx_i^2} > 0 \) where \( x^*_j \equiv m_i(x_i) \) is the solution to \( \frac{\partial g^i(x_i,x_j)}{\partial x_j} = 0 \) where \( g^i \) is the leader’s cost function.

**Lemma 2.** (Rosen, 1965) A simultaneous game between two players who want to minimize their own cost functions has a unique solution if the symmetric matrix \( J + J^T \) is positive definite, where \( J \) is the Jacobian matrix for the first derivative of cost functions.

Based on Lemma 1 and Lemma 2, we provide sufficient conditions of existence and uniqueness of the equilibrium in the hierarchical game. For notational simplicity, we will omit the parameter \( p \) because it does not affect the analytical properties below.

**Corollary 1.** Recall that the hierarchical game [GameSum] defined in Equation 3.10 is a tri-level game and a simultaneous game. The equilibrium is \( (c^*_i(HH), c^*_i(HH), c^*_i(AH), c^*_i(AH), k^*) \) where \( c^*_i(AH) \equiv m_i(c^*_i(AH)) \) and \( c^*_i \equiv m_i(k^*) \). In the tri-level game, the law maker’s equilibrium is denoted by \( k^* \), the AV manufacturer’s equilibrium by \( c^*_i(AH) \) and human drivers’ equilibrium by \( c^*_i(AH) \), respectively. Cost functions of players in the tri-level game are \( SC(c^*_i(HH), c^*_i(HH), c^*_i(AH), c^*_i(AH)) \) and \( c^*_i(AH) \) respectively. In the simultaneous game, human drivers’ equilibrium is denoted as \( (c^*_i(HH), c^*_i(HH), c^*_i(AH)) \). Cost functions of players in the simultaneous game are \( C^*_h(c^*_i(HH), c^*_i(HH), c^*_i(AH), c^*_i(AH)) \), \( i = 1, 2 \). Sufficient conditions of existence and uniqueness of \( (c^*_i(HH), c^*_i(HH), c^*_i(AH), c^*_i(AH), k^*) \) are:

1. \( \frac{\partial^2c^*_i(AH)}{\partial(c^*_i(AH))^2} > 0 \).

2. \( \frac{\partial^2c^*_A(c^*_i(HH),c^*_i(HH))}{\partial(c^*_i(AH))} > 0 \) where \( c^*_i(AH) \equiv m_i(c^*_i(AH)) \) is the solution to \( \frac{\partial c^*_A(c^*_i(HH),c^*_i(HH))}{\partial(c^*_i(AH))} = 0 \).

3. \( \frac{dSC(m_i(k),m_i(m_i(k)))}{dk} > 0 \) where \( c^*_A \equiv m_i(k) \) is the solution to \( \frac{\partial c_A(c^*_i(HH),c^*_i(HH))}{\partial(c^*_i(HH),c^*_i(HH))} = 0 \).

4. \( J + J^T \) is positive definite, where \( J = \begin{bmatrix} \frac{\partial^2c^*_i(HH)}{\partial(c^*_i(HH))^2} & \frac{\partial^2c^*_i(HH)}{\partial(c^*_i(HH))} \\ \frac{\partial^2c^*_i(HH)}{\partial(c^*_i(HH))} & \frac{\partial^2c^*_i(HH)}{\partial(c^*_i(HH))^2} \end{bmatrix} \).

**4.2. Algorithm Design**

A computational algorithm is developed leveraging a special structure in the proposed hierarchical game [GameSum], that is, the law maker’s strategy, denoted by \( k \), implicitly impacts the social cost function via the care levels of AVs and HVs. Thus the upper level game and the game between human drivers and the AV manufacturer can be decoupled once the value of \( k \) is fixed. Accordingly, the rationale of the algorithm is that, given an AV penetration rate \( p \), we first fix the law maker’s decision variable \( k \) in the upper level game, and then solve the equilibrium care levels for the AV manufacturer and human drivers in the mid-level game. By iterating \( k \) in a sufficiently large regime, we will pick the one that attains the minimum social cost for the law maker as its optimal strategy.
Algorithm: Hierarchical Game

Input: Payoff Functions: $C^{(HH)}_{H_1}, C^{(HH)}_{H_2}, C^{(AH)}_{H}, C_{A}, SC$

Output: Players’ Strategies: $k^*, c^{(HH)}_{H_1}, c^{(HH)}_{H_2}, c^{(AH)}_{A}, c^{(AH)}_{H}$

Initialization: $k = 0.1$, $\min = 10^5$

while $k \in C_k$

for $c_A \in C_{c_A}$ do

$c^{(AH)}_{H} = \arg \max_{c_A} c^{(AH)}_{H}(c_A, c^{(AH)}_{H}, k)$

if $\frac{\partial C_A(c^{(AH)}_{H}(c_A), c^{(AH)}_{H}, k)}{\partial c_A} = 0$ then

$c^*_A(k) = c_A$, $c^{(AH)}_{H}(k) = c^{(AH)}_{H}$; // Stackelberg Game between AV manufacturer and HVs

end

end

for $c^{(HH)}_{H_1} \in C_{c_{H_1}}$ do

$c^{(HH)}_{H_2} = \arg \max_{c^{(HH)}_{H_2}} c^{(HH)}_{H_2}(c^{(HH)}_{H_1}, c^{(HH)}_{H_2})$

if $\frac{\partial C_{H_1}(c^{(HH)}_{H_1}, c^{(HH)}_{H_2})}{\partial c_{H_1}} = 0$ then

$c^{(HH)}_{H_1}(k) = c^{(HH)}_{H_1}$, $c^{(HH)}_{H_2}(k) = c^{(HH)}_{H_2}$; // Simultaneous Game between HVs

end

end

if $SC(c^{(HH)}_{H_1}(k), c^{(HH)}_{H_2}(k), c^{(AH)}_{A}(k), c^{(AH)}_{H}(k)) < \min$ then

$\min = SC; k^* = k$

$c^{(HH)}_{H_1} = c^{(HH)}_{H_1}(k)$, $c^{(HH)}_{H_2} = c^{(HH)}_{H_2}(k)$, $c^{(AH)}_{A} = c^{(AH)}_{A}(k)$, $c^{(AH)}_{H} = c^{(AH)}_{H}(k)$; // Law Maker

end

end

5. Numerical Examples

To investigate the behaviors of human drivers, the AV manufacturer, and the law maker in the AV-HV mixed traffic, numerical examples are provided as the AV penetration rate increases. The goals of these examples are to understand: (1) under what conditions human drivers develop a moral hazard, (2) under what conditions the AV manufacturer’s cost in the pure AV market is lower than that of the AV-HV mixed market, and (3) under what conditions an optimal liability rule reduces total social cost. The outcomes are evaluated on three performance measures, namely, the care levels of AVs determined by the AV manufacturer and that of HVs, road safety in light of total crash rate, and social welfare. Accordingly, we would like to test the following hypotheses as the penetration rate of AVs grows:

1. Human drivers take advantage of the AV manufacturer’s choice on AVs’ care level.

2. The AV manufacturer chooses a lower care level for AVs in a pure AV market than in a mixed one.

3. A strategic law maker performs better than a non-strategic law maker in reducing social cost.

We will first fix the liability rule (i.e., assuming that the law maker is not a strategic player and $k = 1$) in Sections 5.1 and 5.2. A baseline model is first tested to investigate how road users and the AV manufacturer may change their care levels and its impact on road safety and social welfare, as the number of AVs increases. Then sensitivity analysis is performed over a set of cost and crash related parameters. In Section 5.3, we relax our assumption and allow the law maker to select an optimal set of liability rules. We will then compare the social cost when the law maker is a non-strategic player and a strategic player, to demonstrate the critical role the law maker plays in the AV-HV mixed traffic.
The parameters, cost functions, and crash probability functions used in the numerical examples are listed in Table 3. Note that the parameters in the payoff functions are carefully selected to fulfill the existence and uniqueness criteria of equilibria in Corollary 1.

<table>
<thead>
<tr>
<th>Name</th>
<th>Notation</th>
<th>Function Form</th>
<th>Parameter</th>
<th>Parameter Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Precaution cost</td>
<td>$S_A(c_A)$ $S_H(c_H)$</td>
<td>$\frac{1}{a c_A - 1} - 1$ $\frac{1}{b c_H - 1} - 1$</td>
<td>$[\alpha; \beta]$</td>
<td>[0.3; 0.5], [0.4, 0.5]</td>
</tr>
<tr>
<td>Crash probability</td>
<td>$P(c_A, c_A)$ $P(c_H, c_H)$</td>
<td>$\frac{a c_A + b c_A + 1}{h c_H + h c_H + 1}$</td>
<td>$[\alpha; h]$</td>
<td>[20; 20], [10; 20], [30; 20], [20; 10], [20, 30]</td>
</tr>
<tr>
<td>Weighting coefficient</td>
<td>$[w_l; w_a; w_h]$</td>
<td>-</td>
<td>-</td>
<td>[0.1; 0.5; 1], [0.1; 1; 1], [0.1; 10; 1]</td>
</tr>
</tbody>
</table>

Table 3: Parameter settings

The proposed algorithm is implemented in Matlab R2017a.

5.1. Game Between Humans and the AV Manufacturer

In the game between humans and the AV manufacturer, a base model is developed where $T = 6$, $a = b = 20$, $\alpha = 0.4$, $\beta = 0.5$, $w_l = 0.1$, $w_a = 1$, $w_h = 1$. We will vary $p$ and test the impact of a growing AV penetration rate on the system performance.

We first plot HVs’ and AVs’ cost functions with respect to care levels in Figure 2(a) to validate our Assumption 3 that AVs can achieve the same care level with a lower cost than humans. The blue solid line is HVs’ precaution cost while the red dashed line is AVs’ sensor cost. We can see two trends: (1) humans require higher cost than AVs to achieve the same care level on the x-axis, and (2) AVs’ care level (determined by sensor specification) can be much larger than that of humans (Kalra and Paddock, 2016; Fagnant and Kockelman, 2015), demonstrating the fact that AVs have much stronger perception and reaction power than humans (Chatterjee, 2016).

Given the care level cost functions, we can then compute the equilibrium care levels for HVs and AVs. Figure 2(b) illustrates how human drivers and the AV manufacturer adjust equilibrium care levels (y-axis) as AVs’ penetration rate $p \in (0, 1)$ (x-axis) increases. Human’s care levels in the $AH$ scenario and the $HH$ scenario are indicated by a blue line and a dashed green line, respectively. AVs’ care level is represented...
by a red line with triangle markers. We first focus on the trend of care levels for HVs and AVs in the \textit{AH} scenario. As the penetration rate $p$ increases, AVs' care level first increases and then slightly decreases after $p > 0.8$. HVs' care level first increases till $p = 0.1$, start decreasing, and then slightly increases again after $p > 0.8$. When $0.1 < p < 0.8$, as the number of AVs increases, the AV manufacturer becomes more attentive while HVs become less attentive. Because the growing number of AVs on the road increases the crash loss of the AV manufacturer, to reduce the total crash loss, the AV manufacturer has to improve safety specification of AVs to increase the care level. When AVs become more attentive, human drivers begin to lower their care levels by taking advantage of increasing AVs' care level. This trend is usually observed in a leader-follower game (Pedersen, 2001). When the leader - AVs - execute higher precaution, the follower - HVs - lower precaution accordingly. Note that HVs' care levels in the \textit{HH} scenario remain constant, regardless of AVs' penetration rate. We can see that HVs execute higher care levels to other HVs than to AVs. According to Definition 3.1, moral hazard happens because human drivers execute less care when encountering AVs than when meeting other HVs. When $p < 0.15$, the blue line is higher than the red line, showing that the AV manufacturer chooses a low care level for the reason that there is a small number of AVs on public roads, not incurring many crashes.

Nevertheless, the AV manufacturer cannot always increase its safety specification, because its cost increases exponentially as sensor specifications increase. To balance total cost, the AV manufacturer has to select relatively cheaper sensors when $p > 0.8$. This is because when AVs dominate the market and the portion of human drivers becomes smaller, the overall traffic environment is improved and thus the sensor cost dominates the total crash loss. Reducing sensor cost slightly would not increase crash loss that much.

We also notice two interesting trends when HVs dominate the market, i.e., $p < 0.1$. First, AVs' precaution is lower than that of HVs. When there are fewer AVs in the market, HVs' crash loss involved with other HVs dominate their cost, so they would drive more carefully when they encounter other HVs. Second, AVs' and HVs' care levels share the same increasing trend as the number of AVs grows. With a smaller AV fleet, the AV manufacturer's precaution cost dominates its crash loss, so a lower precaution level is chosen for AVs. As AVs' number increases, more crashes may happen for AVs, so higher precaution level is preferable. HVs also increase their care levels, possibly because of increasing encounters with an increasing number of AVs.

In summary, HVs and AVs' care levels are correlated in the leader-follower game: when $p < 0.1$, both AVs and HVs gradually increase their care levels; when $p > 0.1$, AVs and HVs' care levels have a negative correlation. In other words, when the AV manufacturer increases AVs' care level, human drivers tend to be more attentive when there is a small number of AVs on public roads and decrease their care levels as the number of AVs increases.
In Figure 3(a) and Figure 3(b), blue lines with triangle markers represent the performance measures for the AA scenario, red lines represent those for the AH scenario, and the yellow dotted lines represent those for the HH scenario. When the AV penetration rate increases, crash probability for both the AA and AH scenarios decreases. The crash probability for the HH scenario remains constant, because one HV’s care level when facing other HVs remains constant regardless of the AV penetration rate. In Figure 3(b), the crash rate for the AA scenario increases, that for the HH scenario decreases, and that for the AH scenario increases first and then decreases. The first two trends are reasonable because as AVs increase and HVs decrease, the average crash rate between two AVs increases while that between two HVs decreases. The interesting one is the AH scenario. The maximum crash rate is attained around $p = 0.5$, i.e., half AVs and half HVs, when traffic exhibit the highest heterogeneity and there are the most encounters between AVs and HVs. The difference in care levels lead to higher crash rate between AVs and HVs. In either an HV-dominated (i.e., $p < 0.5$) or an AV-dominated market (i.e., $p > 0.5$), crash rate is lower because vehicular encounters happen mostly between the same types of vehicles.

In Figure 3(c), both total crash rate, represented by a green line with triangle makers and social cost, represented by a maroon line, have a decreasing trend, implying that the growing number of AVs improves road safety and social welfare. We also compare social cost and total crash rate with those in the pure AV market. In the pure AV market, social cost is represented by a dashed maroon line, and total crash rate is
represented by a dashed green line with triangle markers. Note that in the pure AV market, these two values are only attained when \( p = 1 \). We draw a constant line over \( p \) for the purpose of identifying the critical penetration rate that worsens the system performance in the presence of human drivers. Social cost and total crash rate in the mixed market become smaller than those in the pure AV market when \( p \) is greater than 0.6, 0.5, respectively. It means that the presence of human drivers could jeopardize the system performance. With the AV fleet size growing, the traffic system becomes safer and socially better off.

### 5.2. Sensitivity Analysis

In this subsection, we would like to discuss how parameters in cost and crash related coefficients influence the system outcome, compared to our base model. There are three classes of parameters:

1. **Coefficients** \( \alpha, \beta \) in AVs’ and HVs’ precaution cost functions, respectively: it reflects how much AVs’ (or HVs’) precaution cost changes if AVs’ (or HVs’) care level is increased by one unit. The physical meaning of \( \alpha \) is the marginal production cost of AV sensors. \( \beta \) indicates HVs’ marginal cost of executing a care level, which depends on both drivers perception and reaction time, judgment, maneuvering capability, and vehicles’ characteristics. These two coefficients directly influence care levels and thus the system performance.

2. **Coefficients** \( a, h \) in crash probability functions for three scenarios: it reflects the external environment factors that contribute to the crash probability of AVs (or HVs), given that AVs’ (or HVs’) care level remains the same. The parameter \( a \) (or \( h \)) can be interpreted as the marginal change in the crash probability for AVs (or HVs), if the road environment is improved for AVs (or HVs). For example, government builds more roadside infrastructure that can communicate with AVs so that AVs have better awareness of the environment. For human drivers, the driving environment may become safer because of traffic safety measures such as red light camera enforcement (Retting et al., 2008). A more important environment change to human drivers is the increasing number of AVs. If these AVs create a safer road environment for HVs, the crash probability contributed by HVs can be reduced.

3. **Coefficients** \( w_a, w_h \) in disutility functions of the AV manufacturer and HVs, respectively: It reflects how much weight AVs (or HVs) put in front of precaution cost and crash loss. These weights highly depend on the external liability rule. When the liability rule is fixed, the varying weights could change AVs’ (or HVs’) care level and thus the system performance. Here we mainly discuss the impact of \( w_a \) on traffic performance, because it is crucial to regulate how AV manufactures weigh in traffic efficiency and safety. Human drivers’ weights \( w_h \) should be calibrated from real data and will be left for future research.

#### 5.2.1. Precaution Cost Coefficient for AVs

In the base model, AVs’ sensor cost of precaution cost is defined as \( S_A(c_A) = -\frac{1}{\alpha c_A} - 1 \). We would like to see two variants: (1) coefficient \( \alpha \) is reduced, indicating sensor production cost is cheaper for the same quality, possibly due to government subsidy, and (2) \( S_A(c_A) \) is discounted by a scale factor \( \gamma = \frac{1}{0.1 p + 1} \), demonstrating unit sensor cost reduction, possible due to mass scale effect in economics.

We first compare the case when \( \alpha = 0.3 \) to the base model in Figure 4.
In Figure 4(a), AVs’ precaution cost functions at $\alpha = 0.4, 0.3$ are plotted in a blue and a dashed red line, respectively, as the care level increases. To execute the same care level, the sensor cost with a higher marginal production cost $\alpha$ is larger. Figure 4(b) shows AVs’ and HVs’ care levels in red lines and blue lines, respectively. The solid red line and the solid blue line with triangle markers represent the case when $\alpha = 0.4$ while their corresponding dashed lines represent the case when $\alpha = 0.3$. The trend of AVs’ and HVs’ care levels remains the same as $p$ increases. AVs become more attentive while HVs’ care level is reduced for $\alpha = 0.3$, compared to $\alpha = 0.4$. This is because the AV manufacturer can afford sensors with higher safety specifications at the same cost, thus increasing AVs’ care level. HVs, as followers, take advantage of an increase in AVs’ care level and exhibit moral hazard. Nevertheless, both social cost and total crash rate, represented by solid maroon lines and green lines with markers, respectively, in Figure 4(c), decrease as $p$ increases. More importantly, the performance with $\alpha = 0.3$ is better off that those with $\alpha = 0.4$. In other words, the traffic system is better off with a lower marginal cost of AV production. In summary, if government could subsidy the AV manufacturer (Luo et al., 2019) to reduce production cost, it would greatly encourage the AV manufacturer to increase AVs’ care level and improve the overall traffic system performance.

Figure 5 studies scale effect in economics. This is motivated by the fact that production cost of AVs at the initial deployment stage is prohibitively high, for example, the estimated cost of one AV is $30,000 – 85,000$ (Shchetko, 2014). But it would not remain the same level always. Some estimated that AVs’ cost may drop
to $25,000 – 50,000 each (Fagnant and Kockelman, 2015) due to large-scale production. In Figure 5(a),
the red surface is the discounted precaution cost for AVs, while the blue surface represents the original
precaution cost. They both increase as AVs’ care level increases, but the discounted cost is no greater than
that of the original cost. The original cost (in blue) remains constant along the direction of $p$. We compare
care levels and performance measures in Figure 5(b) and Figure 5(c), respectively. The similar trends as
when we reduce $\alpha$ are observed, because both lower the coefficient $\alpha$ in front of care level and $\gamma$ in front of
precaution cost play the same role in encouraging the AV manufacturer to select higher care levels for AVs,
thus leading to improved system performance. When the scale effect exists, the AV manufacturer can expand
its production scale to meet an increasing demand in the market and thus reduce production cost. Note
that the mass scale effect does not improve social welfare and road safety significantly, probably because of
the small scale discount factor $\gamma$ we choose. But we would like to highlight that, in mass production, AVs’
care level can be improved up to 11% compared to the base line, while total crash rate is reduced by 10% at
maximum. Social cost does not decrease significantly (at maximum 0.9%), because of the increasing cost of
care level.

Figure 5: Sensitivity analysis for the discount factor in sensor cost
5.2.2. External Road Environment

Here we investigate the impact of external environment parameters $a, h$ on the system performance in Figure 6. The upper two figures represent when $a$ is varied while the lower two figures represent when $h$ is varied.

Figure 6: Sensitivity analysis for external road environment parameters

In Figure 6, two lines with diamond markers (one is solid and the other is dashed) represent the scenario when road environment is improved for AVs (or HVs). In the remaining four lines, the solid ones represent the base model and the dashed ones represent the scenario when road environment is worse off for AVs (or HVs). Figure 6(a) and Figure 6(c) plot care levels when varying $a$ and $h$, respectively. Within each case, red lines represent AVs’ care levels while blue lines represent HVs’. Figure 6(b) and Figure 6(d) plot performance measures when varying $a$ and $h$, respectively. Within each case, maroon lines represent social cost while green lines represent total crash rate.

The case with varying the external environment for AVs, i.e., the parameter $a$, is more straightforward to interpret. When the road environment is improved for AVs, in other words, the crash probability for AVs is lower at the same care level, the AV manufacture lowers AVs’ care level because of lower crash loss. Accordingly, HVs’ crash loss in the $AH$ scenario is also lower, thanks to the lower crash probability incurred by AVs. On the other hand, as followers, HVs should increase their care levels when AVs become
less attentive. Driven by two opposite forces, in this case, HVs’ care level also decreases. This indicates that the improved road conditions allow the AV manufacturer and human drivers to drive with lower care levels. Accordingly, both social welfare and road safety are enhanced.

The case with varying the external environment for HVs, i.e., the parameter $h$, is more complicated. When the road environment is improved for HVs, HVs’ care levels increase. Although the crash probability for HVs decreases due to the improve road environment for HVs, it also reduces the crash probability for AVs in the AH encounter scenario. As the leader, the AV manufacturer would reduce its precaution cost by taking advantage of such improvement for HVs. HVs, as followers, have to increase their care levels if the benefit of environment improvement cannot counteract the reduction of AVs’ precaution cost.

5.2.3. Cost Weighting Coefficient in the AV Manufacturer’s Disutility Function

In this subsection, we discuss when the AV manufacturer puts different weights to its precaution cost in a fixed liability scheme, how that will impact the system performance. This discussion is motivated by the existing situation the AV industry faces, that is, many AV manufacturers are unclear to what extent they would be punished if their AVs are involved in any accidents due to uncertainty in the liability system involved with AVs. To minimize the expected loss, most AV manufacturers set their AVs’ driving algorithms highly conservatively. It is interesting to find that Californians have uploaded numerous Youtube videos to share their opinions of how conservatively Waymos drive (Azcentral, 2019). We would like to explore how different cost designs would influence traffic in general and how this impact will evolve as the AV penetration rates grows. To this end, we vary $w_a$ to carry values of 1, 0.5, 10.

![Figure 7: Sensitivity analysis for the weighting coefficient in the AV manufacturer’s cost](image)

Figure 7 shows how the trade-off coefficient $w_a$ in the AV manufacturer’s cost affects AVs’ equilibrium care level and road safety. Two lines with diamond markers (one is solid and the other is dashed) represent the scenario when $w_a = 10$. In the remaining four lines, the solid ones represent the base model ($w_a = 1$) and the dashed ones represent the scenario when $w_a = 0.5$. Figure 7(a) plot care levels when varying $w_a$. Red lines represent AVs’ care levels while blue lines represent HVs’. Figure 7(b) plot performance measures when varying $w_a$. Maroon lines represent social cost while green lines represent total crash rate.

Let us look at $w_a = 0.5$ first, as it is an analogue of the existing situation when AVs act conservatively on public roads. AVs’ care level (represented by the dashed red line) is higher than that of HVs’ (represented by the dashed blue line with triangle markers). The AV manufacturer puts such a high weight on crash loss over production cost that AVs drive attentively. Accordingly, human drivers exhibit moral hazard because they perceive their environment comprised of AVs becomes safer. As the number of AVs increases, the rising rate of AVs’ precaution level gradually slows down, so is the reducing rate of HVs’ care levels. The social welfare and road safety are improved compared to the base model and as AVs grow. One exception is when...
\( p > 0.9 \), social cost at \( w_a = 0.5 \) is slightly higher than that of the base model, due to high precaution cost of the AV manufacturer. Based on above observations, we can say that AVs’ attentive attitude helps improve road safety and social welfare at the beginning with a smaller number of AVs. However, too conservative driving may jeopardize the overall traffic efficiency in the later stage. That is consistent with how California drivers feel while encountering AVs on roads (Azcentral, 2019). A gradual relaxation on \( w_a \) will help improve traffic efficiency in the mixed AV-HV system.

If we raise \( w_a = 10 \), we observe unexpected behavior of AVs. In Figure 7(a), AVs’ care level is much lower than HVs’, because the AV manufacturer needs to reduce production cost as much as they can and care less about crash loss. This makes the AV manufacturer less attentive and accordingly, human drivers have to drive much more careful when they encounter AVs than in the base model. Accordingly, social welfare and crash rate increase first and then decrease, as \( p \) increases. There is a penetration regime when \( p \in (0.1, 0.3) \), social welfare is compromised and road safety is worsened off, because of the increasing vehicle encounters between AVs and HVs. In the pure AV market, both social cost and road safety are higher than those in the pure HV market, indicating that the introduction of autonomous driving technologies, if worse than humans in driving, leads to deteriorating traffic performance. This case may not be realistic, but its take-home message is that, if the AV manufacturer is not regulated in terms of AV technology specifications or is not properly subsidized, the AV manufacturer can be purely profit-oriented and harm the overall traffic system.

### 5.3. Law Maker’s Decision

In the previous examples, the liability rule, i.e., the care level standard ratio \( k \), is fixed. In this part, we assume the law maker is a strategic game player who optimizes \( k \) as the penetration rate of AVs increases.

As we enumerate all the \( k \)'s, the social cost can be plotted as a function of \( k \). Figure 8(a) demonstrates how the social cost changes as \( k \) varies. Each curve represents a social cost function with respect to \( k \), given \( p \). The optimal care level standard ratio \( k^* \) is obtained at the minimal of a social cost curve. We also solved optimal \( k^* \)'s using our proposed algorithm and plot it against \( p \) in Figure 8(b). The solid line represents the solved optimums \( k^* \) and the dashed line represents \( k = 1 \) in the base model. We can see the solved optimums in Figure 8(b) are consistent with those identified from Figure 8(a). As the AV penetration rate increases, the law maker decreases its care level standard ratio first at a faster rate and then at a slower rate. When \( p > 0.5 \), the optimal care level standard ratio is below one, indicating that the law maker sets a much lower standard for human drivers as the AV population grows.

![Figure 8: Law maker’s decision when varying \( p \)](https://ssrn.com/abstract=3509569)
the law maker is strategic and the dashed lines represent care levels in the base model. Red lines represent AVs’ care levels while blue lines represent HVs’. When \( p < 0.5 \), AVs’ care level under the strategic law-maker is higher than that under the non-strategic law maker, and HVs’ care levels under the strategic law-maker is lower than that under the non-strategic law maker. The strategic law maker punishes AVs more than the non-strategic law maker when the AV penetration rate is relatively small, so the AV manufacturer has to increase its care level. Accordingly, HVs take advantage of AVs’ increasing care level and exhibit moral hazard. When \( p > 0.5 \), the trend is reversed, because as the number of AVs increases in traffic, the strategic law maker punishes HVs more than AVs, leading to less attentive AVs. In Figure 9(b), the solid lines represent performance measures when the law maker is strategic and the dashed lines represent performance measures in the base model. Maroon lines represent social cost while green lines represent total crash rate. Social welfare is lower under the strategic law-maker, but total crash rate is slightly higher under the strategic law-maker when \( p > 0.5 \). Because the goal of the strategic law-maker is to minimize social cost, not crash rate, it may compromise road safety a bit so that advanced transportation technologies can be adopted.

![Care Level and Performance Measures](https://ssrn.com/abstract=3509569)

Figure 9: Strategic law maker versus non-strategic law maker (\( k = 1 \))

We then compare how varying \( k \) below or above optimal values influences care levels and the system performance. When the law maker is non-strategic, we assume the liability policy is given, which are \( k = 0.2 \) and \( 5 \), respectively. In each sub-figure of Figure 10, two lines with diamond markers (one is solid and the other is dashed) represent the scenario when \( k = 0.2 \). In the remaining four lines, the solid ones represent the scenario with the optimal liability rule and the dashed ones represent the scenario when \( k = 5 \). Figure 10(a) plot care levels in each case. Red lines represent AVs’ care levels while blue lines represent HVs’. Figure 10(b) plot performance measures in each case. Maroon lines represent social cost while green lines represent total crash rate.

When \( k = 0.2(<1) \), AVs have high care levels while human drivers always choose low care levels. This is because the non-strategic law maker punishes AVs more than humans so that humans take advantage of the AV manufacturer when AVs’ care level goes up. When \( k = 5(>1) \), human drivers have a higher care level standard than AVs, leading to higher care levels for human drivers. As \( p \) increases, human’s care level goes down while that of AVs goes up. Humans also take advantage of AVs’ increasing care levels. With the strategic law maker, the optimal care level for humans is also inversely correlated to that for AVs.

Under the strategic law maker, human drivers tend to have higher care levels than under the non-strategic law maker. This indicates that the non-strategic law maker punishes human drivers to a lesser degree. AVs’ care level under the strategic law maker is between those under the non-strategic law maker. In other words, the non-strategic law maker punishes AVs more than necessary if \( k < 1 \), forcing the AV manufacturer to select a quite high care level in order to reduce crash losses. This stringent policy can ensure traffic safety.
in the mixed traffic but may halt the development of technologies at the initial stage.

Figure 10: Non-optimal care level standard ratios in comparison with the optimal one

6. Conclusion and Future Work

This paper investigates the strategic interactions between AVs and HVs using a hierarchical game and provides liability implications for the mixed traffic. The interactions between vehicles on roads can be categorized into three types of vehicular encounters: HV-HV, HV-AV, and AV-AV. Among them the HV-HV encounter is modeled as a game where each human driver selects her care level. AVs’ selection of care levels while interacting with other AVs and HVs is determined by the AV manufacturer, once a one-time investment of sensors is fixed. To model the role the AV manufacturer plays in the design of autonomous driving, a Stackelberg game between the AV manufacturer and HVs is formulated. On the upper level, the law maker decides an optimal combination of driver liability and products liability rules to regulate both human drivers’ care level selection in presence of AVs and the AV manufacturer’s design of AVs’ care level. The hierarchical game helps us to understand the human drivers’ moral hazard, the AV manufacturer’s impact on traffic safety, and the law maker’s adaptation to the new transportation ecosystem. The game and its algorithm are tested on a set of numerical examples, offering insights into behavioral evolution of AVs and HVs as the AV penetration rate increases and as cost or environment parameters vary. The outcome of the developed game provides an analytical tool to identify optimal liability rules to ensure social welfare and road safety. We find that an optimally designed liability policy is crucial to help prevent human drivers from developing moral hazard and to help the AV manufacturer with a trade off between traffic safety and production costs.

Albeit novel, this work can be extended in the follow ways: (1) In the real world, vehicle encounters may become complicated and may involve more than two vehicles. We will model vehicle encounters in specific contexts, such as rear-end crash, and study the impact of liability rules on different accident types. Also, we assume all accidents share a same crash loss, without distinguishing crash severity, which should be a decreasing function of care level. We will also relax this assumption to consider a hierarchical structure of accident types. (2) We will consider more than one AV manufacturer and assume each AV manufacturer produces more than one AV type characterized by sensor specifications. (3) We will relax our assumption that human drivers become only more aggressive while encountering AVs. Instead, human drivers will drive with less care levels overall as AVs’ penetration rate gradually increases. This relaxation may further reduce overall traffic safety and requires more attention to regulate human’s driving behavior. (4) We will relax our assumption that human drivers have complete information about car scenarios. Instead, human drivers
may have incomplete information about car scenarios. In this case, mixed Nash equilibrium for human’s behavior should be taken into consideration. (5) If the equilibrium of the lower level game is non-unique, the law maker has to make a decision with certain risk-taking attitudes. We will discuss how various risk-taking attitudes will influence the system performance.

7. Acknowledgements

The authors would like to thank Data Science Institute from Columbia University for providing a seed grant for this research.

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