Probabilistic Compliance

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Probabilistic Compliance

Alex Raskolnikov†

Uncertain legal standards are pervasive but understudied. The key theoretical result showing an ambiguous relationship between legal uncertainty and optimal deterrence remains largely undeveloped, and no alternative conceptual approaches to the economic analysis of legal uncertainty have emerged. This Article offers such an alternative by shifting from the well-established and familiar optimal deterrence theory to the new and unfamiliar probabilistic compliance framework. This shift brings the analysis closer to the world of legal practice and yields new theoretical insights. Most importantly, lower uncertainty tends to lead to more compliant positions and greater private gains. In contrast, the market for legal advice tends to reduce compliance over time—a trend that a regulator may counter either by clarifying the law or by reiterating the law’s continuing ambiguity. If detection is uncertain, the probabilistic compliance framework reveals why, contrary to the prevailing view, the standard damages multiplier should be used to counter detection uncertainty but not legal uncertainty. The Article also reconciles economists’ and lawyers’ understanding of probabilities, highlights the challenges of modeling risk-bearing costs resulting from uncertain legal commands, and provides theoretical support for gain-based sanctions beyond the limited settings where the complete deterrence theory has justified their use thus far.

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† Wilbur H. Friedman Professor of Tax Law, Columbia Law School. © 2017, Alex Raskolnikov. I am grateful to James Alm, Scott Baker, Yehonathan Givati, Jacob Goldin, Andrew Hayashi, Bert Huang, Avery Katz, Wojciech Kopczuk, Jacob Nussim, David Schizer, Robert Scott, and workshop participants at Columbia University, Harvard University, Hebrew University, Oxford University, Tulane University, University of Pennsylvania, University of Virginia, University of Washington, Washington University at St. Louis, the annual meetings of the American Law and Economics and National Tax Associations, the Tax Club, and the Tax Forum for valuable comments and suggestions. Charles Collier provided excellent research assistance. All mistakes are solely my own.
Yale Journal on Regulation

Vol. 34, 2017

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Introduction

Economic analysis of legal uncertainty has seen little progress in three decades. The breakthrough contributions in the 1980s produced interesting but ambiguous results while relying on strong assumptions. In the following years, scholars have had little success in relaxing the assumptions or in resolving, limiting, or otherwise illuminating the ambiguity. Not surprisingly, legal uncertainty has faded into the background of law and economics scholarship.

In sharp contrast, it has remained of great importance in the world of legal practice. Disagreements about the appropriate level of the law’s certainty are pervasive. They embroil courts and administrative agencies, domestic and international actors. And they have major policy implications.

Examples are easy to find. The United States, for instance, is the only developed nation resisting the adoption of the International Financial Reporting Standards (IFRS) because the Securities and Exchange Commission (SEC) prefers the more certain Generally Accepted Accounting Principles (GAAP) to the less certain IFRS. Regulation of financial derivatives, to take another example, continues to suffer because one principal regulator, the SEC, prefers a

1. See STEVEN SHAVELL, ECONOMIC ANALYSIS OF ACCIDENT LAW 93-97 (1987);
   John E. Calfee & Richard Craswell, Some Effects of Uncertainty on Compliance with Legal Standards,
   70 VA. L. REV. 965 (1984); Richard Craswell & John E. Calfee, Deterrence and Uncertain Legal
2. For a discussion, see infra text accompanying notes 17-23.
3. A rare exception is Giuseppe Dari-Mattiacci, Errors and the Functioning of Tort Liability, 13 SUP. CT.
   ECON. REV. 165 (2005). His findings are discussed below. See infra note 21.
4. See Martin Geiter & Zehra G. Kavame Eroglu, Whose Trojan Horse? The Dynamics of Resistance Against
more certain, rules-based approach while another one, the Commodities Futures Trading Commission (CFTC), favors a less certain, principles-based strategy. The United States Supreme Court and the U.S. Court of Appeals for the Federal Circuit cannot agree on how certain the patent law should be. The highest court insists on vague formulations while the specialized circuit court prefers clear rules.

Even the tax law—the domain often perceived as dominated by bright-line rules—is infused with tension resulting from legal uncertainty. For instance, it took the Internal Revenue Service (IRS) and the Treasury Department ten years and three tries to interpret a key uncertain term related to controversial corporate inversion transactions. These involve U.S. corporations moving offshore, often to reduce their U.S. tax bills. Congress enacted a punitive tax regime for inverting companies, but withheld its wrath if the inverter has “substantial business activities” in its new corporate home. The regulators first interpreted the term “substantial” with a multi-factor test stapled to a safe harbor and illustrated by clarifying examples. They then moved to a multi-factor test with no explanations of any kind. Finally, they abandoned the multi-factor test altogether in favor of a clear rule. This story is hardly unique.

Legal uncertainty is not only pervasive, it is persistent. Numerous uncertain terms remain undefined for years, even decades. Legislators, administrative agencies, and courts infuse the law with new vague standards all


10. See id. at 379-81.

11. See id. at 384-85; Treas. Reg. § 1.7874-3(b) (interpreting the “substantial business activities” test to mean 25% of the employees, employee compensation, assets, and income of the consolidated group of companies).


13. For example, an IRS official admitted in 2011 that uncertainty surrounding the term “securities” used in corporate reorganization provisions has persistent “since long before I was born.” Amy S. Elliott, Proposed Deemed Asset Sale Regs May Get Rewrite, Alexander Says, 131 TAX NOTES 561, 562 (2011). In the same breath, the official revealed that he had no plans to resolve the uncertainty. See id.
the time.\textsuperscript{14} Multi-billion dollar deals go forward or fall apart based on lawyers’ varying interpretations of legally uncertain provisions.\textsuperscript{15} Whether this uncertainty results from incomplete information, insufficient resources, incompetence, or some other reason,\textsuperscript{16} legal uncertainty is a fact of life. So, it is important to understand its consequences.

Does greater legal certainty benefit regulators or frustrate their goals? Are individuals and firms likely to gain from greater certainty or be harmed by it? What are the effects of the market for legal advice—the market that is of great importance for the most significant economic actors? What is the interaction between legal uncertainty and detection uncertainty? The law and economics literature has little to offer in answering these questions. This Article addresses all of them.

Our poor understanding of the consequences of uncertain laws may appear surprising. Legal uncertainty is a basic concept familiar to any law student, not to mention lawyers and judges. Any time the law uses terms like “reasonable,” “substantial,” “significant,” “material,” or “due”—any time, that is, the law relies on a standard rather than a rule—a rational actor taking a legally uncertain position faces a seemingly straightforward tradeoff. The actor stands to derive a benefit if his position is deemed compliant with the law. No bright line separates compliance from noncompliance. The actor may change his behavior—adjust his position—along a certain dimension to increase the likelihood of compliance, but this adjustment is costly. The actor chooses his position by maximizing benefits net of costs. How does legal uncertainty affect the actor’s choice?

\textit{Legal Uncertainty in the Optimal Deterrence Framework}

The path-breaking work analyzing this problem came three decades ago from John Calfee and Richard Craswell (C&C) and, independently, from Steven Shavell. Both C&C and Shavell evaluate the actor’s response to uncertain legal commands by comparing it to a particular reference point. For

\textsuperscript{14} For instance, section 871(m), added to the Internal Revenue Code in 2010, provides that some payments will be treated as “dividend equivalents” (and subject to a highly disadvantageous tax treatment) if they are “substantially similar” to payments described in the statute. Pub. L. No. 111-147, 124 Stat. 71 (2010). \textit{See also} T-Mobile S., LLC v. City of Roswell, 135 S. Ct. 808, 811 (2015) (holding that a city denying a siting application must provide written reasons for the denial “essentially contemporaneously” without specifying what time period satisfies this standard); Rev. Proc. 2015-43, 2015-40 I.R.B. 467 (ruling that the size of “active business” matters for the purposes of satisfying the spinoff provisions, but not specifying what size is acceptable).

\textsuperscript{15} \textit{See} Williams Corp., Inc. v. Energy Transfer Equity, L.P., 2016 WL 3576682 (Del. Ch. June 24, 2016) (concluding that Latham & Watkins’s refusal to deliver a “should” tax opinion was reasonable and allowing Energy Transfer Equity L.P. to withdraw from its proposed $33 billion merger with the Williams Companies); Alison Sider & Austen Hufford, \textit{Williams Files Appeal To Close Energy Transfer Merger}, WALL ST. J. (June 28, 2016) (referring to the original value of the deal as $33 billion), http://www.wsj.com/articles/williams-shareholders-opt-for-ete-deal-1467036532.

\textsuperscript{16} Answering the important question about the causes and persistence of legal uncertainty is not this Article’s objective.
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C&C, this point is the socially optimal level of behavior; for Shavell, it is an exogenous level of care (which is determined with the goal of maximizing welfare or, under some assumptions, efficiency). These reference points are crucial to the analysis. If behavior falls short of the optimal level—say drivers exceed the efficiency-maximizing speed—uncertainty results in costly under-deterrence. If behavior surpasses the optimum, the problem is over-deterrence. Either way, the inquiry aims to understand how varying the vagueness of a speed limit changes the efficiency of the drivers’ behavior.

This optimal deterrence framework is fruitful if we may plausibly estimate the optimal level of behavior. Perhaps we may do so when it comes to speeding. But what about the “substantial business activities” test for corporate inversions? Here, we encounter a very challenging problem. Not only is there no theory of optimal corporate reorganizations, there is no theory of optimal corporate tax. Even the theory of optimal capital income taxation (with corporate tax being one form of it) is not well developed. So we have no way of deciding what level of business activity in the inverter’s new home country is optimal if the goal is to maximize efficiency or welfare. The optimal deterrence theory sheds little light on how best to interpret—or enforce—the “substantial business activities” test or many similar vague thresholds found throughout the Internal Revenue Code. Moreover, tax is not unique. Many (most?) other real-life regulatory regimes do not come close to efficiency maximization.

In all those regimes, the optimal deterrence theory has very limited applications.

The gap between the optimal laws and the real ones limits the policy relevance of C&C’s and Shavell’s foundational results. C&C’s key conclusion that legal uncertainty creates conflicting incentives for over- and under-

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17. See SHAVELL, supra note 1, at 97. While Shavell’s discussion of legal uncertainty does not focus on whether this exogenous level is socially optimal, most of the rest of the book does precisely that. See id. at 7. Thus, his analysis of legal uncertainty adopts the optimal deterrence framework implicitly. Efficiency maximization does not take distributional consequences into account, while welfare maximization does. If one accepts the argument, or simply assumes, that all distributional issues should be addressed by the tax-and-transfer system, one would conclude that legal rules should be designed without accounting for distributional consequences. In that case, efficiency-maximizing legal rules will be welfare-maximizing as well. See, e.g., David Weisbach, Distributionally Weighted Cost-Benefit Analysis: Welfare Economics Meets Organizational Reform, 7 J. LEGAL ANALYSIS 151, 151-52 (2015).

18. One may imagine that a negligence standard at least roughly maximizes efficiency, or perhaps one could have imagined this in the early days of law and economics. Richard Craswell’s more recent views about the social optimality of law are much more ambivalent. See Richard Craswell, In That Case, What Is the Question? Economics and the Demands of Contract Theory, 112 YALE L.J. 903, 911 (2003).

19. For an extended discussion, see Alex Raskolnikov, Accepting the Limits of Tax Law and Economics, 98 CORNELL L. REV. 523, 560-66 (2013).

20. For an argument that the common law is unlikely to produce efficiency-maximizing rules, see Anthony Niblett et al., The Evolution of a Legal Rule, 39 J. LEGAL STUD. 325, 330 (2010). For an expression of similar skepticism about contract law, see Eric A. Posner, Economic Analysis of Contract Law After Three Decades: Success or Failure?, 112 YALE L.J. 829, 830 (2003); Craswell, supra note 18.
deterrence relative to the social optimum is not helpful if we do not know where the social optimum lies. Their conclusion that uncertainty leads to ambiguous predictions of behavior—with actors both exceeding and falling short of the social optimum—is uninformative for the same reason. Shavell’s similar conclusion with respect to the level of due care is likewise unhelpful if we have no idea what that level is. Thus, not only are the key results reached by C&C and Shavell ambiguous, they are difficult to interpret without the social optimality assumption or the exogenously given level of behavior.21

Legal Uncertainty in the Probabilistic Compliance Framework

This Article offers a new take on legal uncertainty by focusing on compliance rather than deterrence. Scholars working in the optimal deterrence tradition use the terms deterrence and compliance interchangeably.22 This Article draws a sharp distinction between the two. It uses the term deterrence to refer to the goal of assuring that an agent’s behavior maximizes social welfare.23 In contrast, it uses the term compliance as a legal rather than economic concept. By compliance, this Article simply means compliance with the law, without making any assumptions about the law’s welfare characteristics. When the law is uncertain, greater compliance means a higher likelihood that one’s position will be viewed as lawful by the relevant authority—a higher probability of success. Compliance with uncertain law, therefore, is necessarily probabilistic.

This change in focus offers greater clarity as a positive matter but greater ambiguity as a normative one. On the descriptive side, the Article’s first notable result is that the famous ambiguity in the relationship between legal uncertainty and optimal deterrence discovered by C&C and Shavell largely disappears when we focus on the interaction between uncertainty and compliance. While greater certainty may lead to either over- or under-deterrence, it generally leads to greater compliance. This conclusion follows from the simulations based on this Article’s model. It also follows from C&C’s original simulations that I replicate. And it holds if a normal distribution (generally used in C&C’s and this Article’s simulations) is replaced by several other plausible probability distributions. To be sure, simulations do not amount

21. Dari-Mattiacci, supra note 3, extends C&C’s analysis but finds the same or even greater indeterminacy in the relationship between uncertainty and deterrence. A few extensions that eliminate this indeterminacy (Dari-Mattiacci’s Propositions 12 and 15) model situations that lack the key feature of the regulatory model offered here and the tort model mostly focused on by Craswell & Calfee—the discontinuous jump in actor’s payoff upon a finding of liability. See id. at 179-83.

22. Consider, for example, the titles of the two C&C articles on legal uncertainty. See Calfee & Craswell, supra note 1; Craswell & Calfee, supra note 1.

23. This is a standard use of the term “optimal deterrence.” See, e.g., Amitai Aviram, Allocating Regulatory Resources, 37 J. CORP. L. 739, 742 (2012).
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to a rigorous proof. But the conclusion that greater certainty leads to greater compliance is robust.

This conclusion is reinforced when one considers the likely effects of the market for legal advice. The Article suggests two effects. First, as lawyers grapple with each new uncertain legal test, they often converge to a common interpretation even when the government issues no guidance. In this model’s terms, this convergence means a decrease in the perceived uncertainty while the actual uncertainty (the content of the legal standard) remains unchanged. Second, clients pressure their counsel to condone progressively lower compliance efforts as sufficient to fulfill an uncertain legal command. This pressure is entirely rational: lower compliance efforts are less costly. But it leads to a one-way ratchet in the absence of government intervention: lawyers interpret uncertain standards as being increasingly forgiving. Both effects yield the same result. Over time, clients take increasingly aggressive, less plausible, less likely to succeed positions.

Another result is that greater certainty typically leads to larger private gains. This would not surprise anyone familiar with the unrelenting efforts by sophisticated firms (that may be plausibly assumed to be risk-neutral) to induce regulators to clarify uncertain standards. The model also predicts that when rational actors take uncertain legal positions that are observed by the regulator, their positions are very likely to have more than a fifty-fifty chance of success. This, too, would not surprise legal practitioners involved in the opinion practice in connection with public deals.

A further descriptive contribution made here is to reconcile a lawyer’s understanding of uncertainty with that of an economist. The two have different origins and result from different mental processes, but they produce the same relationship between the probability of success and the actor’s effort to comply with the law. Thus, the widespread use of the language of probability theory by legal advisors is not only sensible, but is consistent with the economic models relying on that theory to investigate legal uncertainty.

Implications of the Probabilistic Compliance Framework

What should we make of these results? This is when the departure from the optimal deterrence framework becomes an issue. The welfare maximization objective may be unrealistically ambitious, but it is clear, rigorous, and has a significant normative appeal. Without it, how should we evaluate the effects of legal uncertainty just described?

24. Shavell’s results are based on comparative statics rather than simulations, but his approach has other limitations. See infra text accompanying note 119.

One possibility is to posit that law should facilitate compliance. Unfortunately, compliance is a fuzzy objective if law is uncertain. The problem is not that compliance has no clear economic interpretation—the degree of compliance is simply the probability of success. The fuzziness comes from specifying what this probability ought to be. Without insisting on any particular answer, this Article makes four observations.

First, regulators of all stripes generally prefer positions that are more likely to be legal to those less likely to be so. Second, while law enforcers want more compliance, they do not ask for certain compliance—they accept probabilistic compliance without demanding perfection. Third, the law sometimes expressly conditions a favorable outcome on the probability of success exceeding 50%. And fourth, scholars have argued that compliance with uncertain standards should be understood as behavior that is more likely to be legal than illegal. If one finds these arguments appealing, one would be interested in this Article’s insights about the effect of legal uncertainty on compliance.

Another approach is to posit that, if we take the law as given and irreducibly uncertain, it makes sense to maximize private gains of the regulated parties, at least when their behavior is observed by enforcement authorities. If externalities and other market failures are unlikely, or if the social welfare effects of externalities are ambiguous, this may be a plausible evaluative criterion.

This Article does not defend either the compliance or the gain maximization as normatively appropriate. But for those who find compliance or private benefits to be appealing, or policy-relevant, or just intellectually interesting considerations, this Article’s takeaway is that greater legal certainty tends to increase both.

Thus, whether it is the SEC defining the term “control” in terms of specific percentages or a vague standard, or the IRS making a similar decision about the term “substantial,” or the Financial Accounting Standards Board (FASB) choosing whether to require companies to book a reserve when outlays

26. See infra text accompanying notes 111-114.
27. See infra text accompanying notes 148-149.
28. See infra text accompanying notes 115-117.
30. This will be the case if a regulated activity produces an externality (or something similar to it) but the regulation itself is welfare-reducing. For a discussion, see Raskolnikov, supra note 19, at 574.
31. For scholarship using this normative criterion, see infra text accompanying notes 142-143.
32. See, e.g., Gelter & Eroglu, supra note 4, at 116-19 (examining various degrees of specificity of defining “control”).
33. See supra text accompanying notes 7-12.
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are “probable” or to specify this term more precisely,34 or the Department of Justice (DOJ) considering jointly with the Federal Trade Commission (FTC) whether to define relevant markets by referring to “effective competition” or by using numerical thresholds,35 the regulators should be aware that greater legal uncertainty is often detrimental both to private actors and to compliance with the law.

Some evidence suggests that regulators misunderstand these relationships.36 At the same time, there are encouraging examples of regulators responding appropriately to the negative (from the compliance perspective) consequences of the market for legal advice.37

The Article also illuminates the interaction between legal uncertainty and detection uncertainty. C&C investigate the use of the standard damages multiplier in the optimal deterrence setting, assuming that the multiplier is used to reverse both types of uncertainty.38 This Article explains that if policymakers are interested in compliance rather than deterrence, the multiplier should offset detection uncertainty alone. This conclusion is new to the literature, and it has a clear rationale.

Using a multiplier to offset legal uncertainty in the optimal deterrence framework converts, in expectation, a threshold regime (such as negligence39) into a strict liability regime. As is well known, both may be welfare-maximizing.40 Thus, there is no need to fine-tune the use of the multiplier to separate legal and detection uncertainty. In the compliance framework, however, using a multiplier to offset legal uncertainty converts an uncertain legal standard into an outright prohibition. That is because the stronger the position one takes, the greater one’s likelihood of success, the smaller the chance of liability, and the larger the multiplier. No rational policymaker would endorse this result because if the policymaker had preferred an outright prohibition, it would have enacted a clear rule in the first place. Thus, when the

34. See Jennifer L. Blouin et al., Pre-Empting Disclosure? Firms’ Decisions Prior to FIN No. 48, 85 ACCT. REV. 791, 794 (2010) (explaining the switch from the general “probable” standard for booking tax reserves to the 50% threshold).
35. See Richard J. Wegener et al., Nonprice Vertical Restraints Tying, Bundling, Rebates & Loyalty Discounts—Consumer Friendly or Exclusionary Conduct?, 56 ALI-CLE 41, 142-46 (2012) (describing the “small but significant nontransitory increase in price” standard adopted by the Horizontal Merger Guidelines as the “most analytically rigorous approach” to defining relevant markets).
36. See infra text accompanying notes 153-155.
37. See infra text accompanying notes 173-177.
38. See Craswell & Calfee, supra note 1, at 292-93 (assuming that the standard multiplier is the inverse of the probability of punishment, which reflects both detection and legal uncertainty).
39. For a suggestion that the term “threshold regime” is more appealing than “fault-based” regime, see Alex Raskolnikov, Irredeemably Inefficient Acts: A Threat to Markets, Firms, and the Fisc, 102 GEO. L.J. 1133, 1142 (2014).
law is not optimal, the multiplier should account for detection uncertainty alone.

These are the Article’s main results. Along with additional findings, they are presented in three parts. Part I offers the basic model, locates it in the literature, reconciles the meaning of probability statements by lawyers and economists, and offers compliance (probability of success) as the evaluative criterion for studying uncertain standards that may not be plausibly assumed to maximize efficiency or welfare. Part II presents the main results regarding the relationship between uncertainty on the one hand and compliance and private gains on the other. It discusses the effects of the market for legal advice, the design of the damages multiplier, and the use of probabilistic compliance as a plausible regulatory objective. Part III briefly explores complications arising from multi-dimensional legal standards, variable benefit functions, and costs that change abruptly upon a slight shift in the actor’s position. It also identifies the unappreciated challenges of investigating the effects of legal uncertainty on risk-averse actors. Part IV explains why the Article’s findings extend the analysis of gain-based sanctions for violations of non-optimal laws beyond the narrow set of cases traditionally considered by theorists of complete (rather than optimal) deterrence. A brief conclusion follows.

I. The Groundwork

Uncertain legal commands are the stuff of everyday life. Lawmakers enact them, lawyers interpret them, and people make decisions guided by them. Yet there has been surprisingly little investigation of the basic questions related to legal uncertainty, at least in the law and economics literature.

What do lawyers convey when they express various degrees of confidence in a given position? How do courts and lawmakers evaluate compliance with uncertain legal standards? What does it mean to “comply” with a vague legal command in the first place? This Part starts with a simple model and lays the groundwork for its analysis by offering answers to these and related questions.

A. Motivating Examples

To motivate the inquiry, consider a few simple vignettes. Imagine a major U.S. company wishing to sell one of its two businesses to a potential acquirer. If the sale can be structured as a tax-free transaction, the company will realize a benefit of $100 million. If, however, the sale is taxable, the tax liability will erase the benefit entirely, and the sale is not worth pursuing. Tax lawyers inform the company’s CEO that in order to qualify for the tax-free treatment, the company must transfer “substantially all” of its assets to the acquirer.42

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41. See supra text accompanying notes 14-15.
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Moreover, the lawyers explain that the meaning of “substantially all” is uncertain. Transferring no assets clearly fails the test, while transferring all assets definitely satisfies it. For anything in between, the greater the percentage of the assets transferred, the more confident the lawyers are that the test is met. Unfortunately, transferring assets in excess of a certain amount is undesirable for the company, and the greater the excess, the less happy the company’s CEO.

Similar scenarios may be constructed in almost any area of the law. Imagine a corporate board contemplating a potentially profitable transaction that is expected to raise the company value by $100 million. The board must decide how much independent advice it should obtain to protect the transaction from a challenge under the demanding—but vague—“entire fairness” test of the Delaware corporate law. There are plenty of friendly bankers standing ready to provide the “fairness opinions” that the board needs, but the bankers charge hefty fees for those opinions. The more opinions the board receives, the greater the cost to the company, but the greater the likelihood that the court will uphold the transaction against a future challenge.

Or take an administrative agency considering a new regulation that is expected to produce, in the agency’s view, a $100 million benefit. The agency must decide how much of its limited resources to spend on the cost-benefit analysis in order to protect the regulation from being invalidated by a court. The agency has no doubt that the new regulation will turn out to be beneficial no matter how much cost-benefit analysis the agency undertakes. But the analysis is expensive. On the other hand, the more the agency spends, the more thorough the analysis, and the greater the chance of a judicial approval.

Even legislative decision-making can be modeled in the same way. Imagine Congress finalizing a piece of legislation expected to produce $100 million of (what legislators view as) a surplus. The legislature must decide how much “due process” the legislation should provide to its beneficiaries in order to withstand a constitutional challenge. Importantly, greater procedural protections (such as multiple hearings) do not change the surplus but impose a greater cost on the government. Greater protections also increase the likelihood that a constitutional challenge will fail. Other examples may be easily offered, all similar in the key respects discussed next.

B. The Model and Its Context

All of the vignettes just described have important common features. Each vignette involves a rational actor who may be assumed to be risk-neutral. The

45. See U.S. CONST. amend. V.
actor would like to obtain a benefit $b$, but faces uncertainty regarding the benefit’s availability under the law. The actor chooses his position under an uncertain legal standard by choosing the variable $x$. This variable may reflect a percentage (of assets, economic exposure, market share, risk of loss, and the like), a time period (sometimes measured in days, other times in years), a number (of fairness opinions, expert cost-benefit reports, administrative hearings, and so on), or some other dimension. Various values of $x$ (that I will also call the actor’s positions or compliance efforts) correspond to various probabilities of success on review. Assume that a greater compliance effort corresponds to a higher probability of success—that is, to a more compliant, stronger, more conservative legal position.46 Achieving higher values of $x$ comes with a greater cost, however. That is, stronger positions are costlier than weaker ones for private actors.

A few other features are common to all the vignettes. The uncertainty surrounding the availability of the benefit may not be resolved ex ante, such as by acquiring legal advice. After the actor chooses his position $x$, some authority will scrutinize it and either allow the benefit by finding the position to be compliant or deny it by finding the position to be non-compliant. That is, detection and scrutiny are assured. Finally, and provisionally, assume that violators of uncertain legal commands face no sanctions; they only lose the benefit $b$.47

How does the actor choose $x$? He does so by assessing the costs and benefits of taking positions of varying strength. Assume that the cost depends on $x$ in some yet unspecified way, so $C(x)$ is the actor’s cost function. The actor who chooses any particular $x$ incurs the cost $C(x)$ with certainty. In contrast, the receipt of the benefit $b$ is not guaranteed. Moreover, the actor can influence the likelihood of securing that benefit. Specifically, the probability of success depends on the compliance effort, $x$. Thus, the probability of success is not just a number—it is a function of $x$ that we will call $F(x)$. A rational actor will maximize his expected gain, $G(x)$, which is the net gain from exerting the compliance effort $x$ in order to obtain the benefit $b$ with probability $F(x)$ while incurring the cost $C(x)$.48

Having specified the key features of the model, we can now locate it in the literature. The optimization problem offered here is very similar to the one

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46. Nothing of interest would change if we made the opposite assumption. As the later discussion explains, the statement in the text is always true if we hold the level of uncertainty fixed, but not otherwise. See infra text accompanying note 160.

47. This is a very realistic assumption for many standards and a substantial range of $x$ values. For example, many tax positions that lose in court (that is, that lead the taxpayer to lose the tax benefit) are not subject to penalties even if the positions were worse than a fifty-fifty bet to begin with. See I.R.C. § 6662(d)(2)(B)(i) (2012) (stating that understatement of income tax does not lead to penalties if it results from a position having substantial authority). Similarly, the remedy of disgorgement is a quintessential example of the loss of $b$. See, e.g., 15 U.S.C. § 78p(b) (2012) (requiring the disgorgement of short-swing profits by corporate insiders).

48. Formally, the actor’s expected gain is thus: $G(x) = bF(x) - C(x)$. 

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explored by C&C and Shavell. C&C posit an actor such as a driver choosing how fast to drive under an uncertain negligence standard. The actor compares a higher certain benefit of greater speed against a higher uncertain cost of accidents for which the driver may be found liable. In contrast with the present setup, their model has variable (rather than fixed) benefits and uncertainty that relates to losses (rather than gains). Still, the two models have much in common, so it is important to be clear about the differences.

Most essentially, the evaluative framework offered here differs from C&C’s in key respects. C&C assume that, although the law is uncertain, it is either socially optimal or it may be made so. They posit that there is a specific value of \( x \) (I will call it \( x^{so} \)) that corresponds to the most efficient legal regime. Naturally, they focus on how the private actor’s choice of \( x \) compares to the socially optimal one. The closer \( x \) is to \( x^{so} \) the better; \( x \) exceeding \( x^{so} \) means under-deterrence (think of driving too fast) while \( x \) falling short of \( x^{so} \) means over-deterrence. C&C famously conclude that legal uncertainty yields incentives leading to both under- and over-deterrence with an optimal legal standard and that both phenomena arise in their simulations. Shavell reaches a similar result without relying on simulations by modeling legal uncertainty as an error in determination of an exogenously given (though not necessarily optimal) level of due care.

As discussed in the Introduction, the social optimality assumption is difficult to maintain in many legal regimes. We are unlikely to know—possibly in theory, and certainly in practice—the welfare-maximizing amount of assets to be transferred in a tax-free spinoff, or the socially optimal number of fairness opinions, or the welfare-maximizing delay in issuing a written explanation for denying a government permit. A further difficulty arises because in any regulatory regime that involves transfers (taxation being an obvious example), the key result characterizing the social optimum—the result on which the optimal deterrence theory and C&C’s analysis heavily rely—does not hold. If we drop the social optimality assumption, C&C’s formal analysis no longer applies and their evaluative criterion is no longer available. Shavell does not

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49. This Article refers primarily to C&C’s analysis because it is more informative for the reasons discussed below. See infra text accompanying note 119. I discuss Shavell’s work where its findings or implications differ from those of C&C.

50. Thus, C&C’s optimization problem is \( G(x) = B(x) - C(x)F(x) \). See Craswell & Calfee, supra note 1, at 281.

51. See SHAVELL, supra note 1.


53. If, for instance, the cost imposed on the private actor by a legal regime is a transfer, it is no longer true that the social optimum corresponds to the point where the marginal private cost is equal to the marginal private benefit, as is true for regimes intended to force private actors to internalize externalities.

54. The formal analysis does not apply because in the absence of the social optimality assumption, it is impossible to derive C&C’s first-order condition by taking advantage of the fact that at
rely on the social optimality of the standard of care for his analysis of uncertainty, but he takes this standard as a given. In contrast, when lawyers consider an uncertain legal test such as “substantial business activity” or “substantially all,” no particular number is available as an obvious reference point.\footnote{55} This Article offers a new interpretation of the model by shifting the focus from optimal deterrence to probabilistic compliance. This shift yields new results about the interaction between uncertainty and compliance, the effect of uncertainty on private gains, and the interplay between legal and detection uncertainty, to name a few. Notably, some of the findings arise not only from the present model, but also from replicating and reinterpreting C&C’s original simulations as well.

C&C’s work has not been the only inquiry into the efficiency properties of uncertain law. Similar to this Article (and in contrast with C&C), these other contributions do not assume the existence of socially optimal legal rules. However, the uncertainty they investigate differs from the one studied here (and by C&C and Shavell) in key respects.

Some scholars model uncertainty as arising from possible future legislative changes.\footnote{56} Obviously, an individual actor may not adjust his choices under current law (as the present model allows) in order to influence his probability of success under some future yet unknown rule. Another approach views uncertainty as the actor’s lack of knowledge of legal consequences. These models allow actors to reduce or eliminate their ignorance by learning the rules or acquiring advice.\footnote{57} In contrast, the legal uncertainty studied here is irreducible. In fact, acquiring advice may increase it.\footnote{58}

Several authors model tax law uncertainty as a random variation of taxable income around the mean. Some of these models do not allow the actor to vary the outcome by choosing different values of $x$—the key feature of the

\begin{footnotes}
55. For a suggestion on such a number, see infra text accompanying notes 103-104; for discussions of the weakness of this suggestion, see infra the concluding paragraphs of Sections I.E and II.D.


58. See Boris I. Bittker & Lawrence Lokken, FEDERAL TAXATION OF INCOME, ESTATES AND GIFTS ¶ 22.6.3 (2005) (discussing the controversy regarding the “principal place of business” term that preceded and followed the Supreme Court’s Soliman decision).
\end{footnotes}
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present model. Others are tax-specific. They incorporate uncertainty into the optimal tax theory or rely on unique features of the tax law.

A few investigations come closer to the present one in interpreting legal uncertainty, but they lack at least one of the key features of the proposed model. David Ulph investigates taxpayers who face uncertain legal commands (in the sense discussed here) and purchase tax schemes of various aggressiveness. However, one of his model’s main drivers—the risk of retroactive legislation—is not a serious concern in most legal settings. Paul Beck and co-authors study legal uncertainty and variation in tax reporting aggressiveness. Kate Krause models a parameter that may be interpreted as the strength of one’s legal position as well. Lillian Mills and co-authors present a model where taxpayers may take positions of varying strength. And Michael Graetz and co-authors investigate the effect of tax advice on decisions of taxpayers facing different probabilities that their deductions would be disallowed (which the authors call “exposure”). None of these models, however, treat the actor’s compliance effort as an endogenous variable. In contrast, the fundamental question that the current model investigates is how a rational actor facing uncertain law chooses his compliance effort.

Economic models featuring an endogenous parameter similar to this model’s do exist. Although they do not interpret it as the compliance effort, this does not necessarily make them inapplicable. Shlomo Yitzhaki considers

59. See Woon-Oh Jung, Tax Reporting Game Under Uncertain Tax Laws and Asymmetric Information, 37 ECON. LET. 323, 323-24 (1991) (posing that “a taxpayer does not know her true taxable income due to tax law complexity, [but] she is assumed to be privately informed of its distribution”).

60. See Kaplow, supra note 57, at 69 (connecting his analysis of taxable income uncertainty to the optimal tax theory and recognizing the resulting complexity); Suzanne Scotchmer & Joel Slemrod, Randomness in Tax Enforcement, 38 J. PUB. ECON. 17, 19 (1989) (basing the model on the fact that tax understatements are subject to penalties but tax overstatements are not rewarded by the government).


62. See id. at 8-20. For a discussion of this and other limitations of Ulph’s analysis, as well as a discussion of other scholars’ contributions mentioned in this paragraph, see Alex Raskolnikov, Six Degrees of Graduation: Law and Economics of Variable Sanctions, 43 FLA. ST. L. REV. 1015, 1027-28 (2016).


64. See Krause, supra note 57, at 400.


67. Mark Cronshaw and James Alm offer a model that includes a parameter (α), defined as “the probability that a high-income taxpayer reports low (i.e., the probability of noncompliance or cheating).” Mark B. Cronshaw & James Alm, Tax Compliance with Two-Sided Uncertainty, 23 PUB. FIN. Q. 139, 144 (1995). Importantly, the taxpayer in the model is free to choose the value of α, making it endogenous. It is unclear, however, what this probability corresponds to in real life—that is, how a taxpayer may choose this probability.
the probability of detection that depends on the amount of evaded income chosen by the taxpayer. 68 Louis Kaplow studies the probability of detection that varies with the taxpayers’ expenditures on concealing their evasion. 69 However, neither model links the endogenous choice of x to a probability distribution of the kind considered here. 70 Therefore, neither model sheds light on how this distribution affects the choice of x—the central question of this Article.

Overall, the decades-old contributions by C&C and Shavell remain the leading economic inquiries into rational decision-making under uncertain legal commands. 71 Therefore, this Article uses their findings as reference points.

C. The Meaning of Probabilities

The basic model presented in the previous section has only three variables. Two of the three require little elaboration. The benefit b is assumed to be fixed, and the cost C(x) reflects the cost of compliance, whatever it may be. The meaning and shape of the probability distribution F(x), however, is far from obvious.

C&C spend little time on investigating the source of the distributions they use and the meaning of probability statements they make. In their first article, they introduce three bell-shaped curves as “three possible distributions of probabilities” 72 without further elaboration. In their later, more formal piece they assume a “probability density function associated with the uncertain legal standard.” 73 Shavell says even less about the probability distribution he studies. 74 These generic specifications are standard and uncontroversial for legal economists. But they have perplexed or troubled lawyers and academics

70. Neither Yitzhaki nor Kaplow discuss the likely shape of the probability distribution, and there appears to be no reason to assume that the relationship between the probability of detection and either the amount of evaded income (in Yitzhaki) or the concealment efforts (in Kaplow) has the shape that is posited here; nor is either distribution bounded like the present one. In fact, it appears plausible to assume that increasing the endogenous parameter (the amount evaded or the concealment effort) yields diminishing marginal returns, producing a concave density function in contrast with an S-shaped one discussed in the next section.
71. For recent work relying on C&C’s analysis of legal uncertainty with an implicit assumption that this analysis reflects our best understanding of the subject, see Daniel Shaviro, Disclosure and Civil Penalty Rules in the U.S. Legal Response to Corporate Tax Shelters, in TAX AND CORPORATE GOVERNANCE 229, 241 (Wolfgang Schön ed., 2008); Sven Hoeppner & Laura Lyhs, Behavior Under Vague Standards: Evidence from the Laboratory (JENA Economic Research Paper #2016-010).
72. Calfee & Craswell, supra note 1, at 971
73. Craswell & Calfee, supra note 1, at 281 n.6.
74. For a discussion, see infra text accompanying notes 105-106.
interested in legal uncertainty as it actually exists. Therefore, we take up the task of demystifying $F(x)$.

Thus far, we have only said that $F(x)$ reflects the uncontroversial assumption that the probability of success depends on the compliance effort. The more assets the corporation transfers, the more likely it is to meet the “substantially all” tax standard. The more fairness opinions the board of directors obtains, the more likely it is to satisfy the “entire fairness” test. What more can be said about $F(x)$? And how can the answer be connected to the basic probability theory? To address these questions, we begin by reflecting on how legal advisors interpret uncertain standards.

To start, note that in many cases, $x$ is bounded. In terms of the model, it varies between 0 and 1. That is because no matter how vague the standard, lawyers know that some compliance effort satisfies the standard for certain, and some other level of effort flunks it just as surely. For example, lawyers may conclude that transferring 0% (or 20%, or even 40%) of the assets definitely flunks the “substantially all” test while transferring 100% of the assets (or 95%, or even 90%) certainly satisfies it. Or lawyers may believe that a single fairness opinion is certainly insufficient under the “entire fairness” test while a dozen opinions is surely enough. To take another example, lawyers may opine that a two-day gap definitely satisfies the “essentially contemporaneous” threshold while a month-long delay certainly does not. In any of these cases, the lower bound of $x$ (producing certain noncompliance) corresponds to zero and the upper bound of $x$ (producing certain compliance) corresponds to one. To simplify the exposition, assume that lawyers believe that the “substantially all” standard is surely not met if a taxpayer transfers no assets at all and is surely met if the taxpayer transfers all assets. So the [0,1] range for $x$ corresponds to 0% to 100% of the assets.

Within the 0-to-1 (or 0% to 100% of the assets) range, lawyers’ beliefs typically have the following structure. First, a lawyer would be prepared to tell the client what he views as the best interpretation of the “substantially all” standard or any other standard if he had to pick a single number. To make the matter more concrete, let us assume that this number is 0.6 (or 60% of the assets). Moreover, the legal advisor would typically explain that there is a relatively small range of percentage values that likely describes the uncertain standard in question. Let us say that this range is between 0.5 and 0.7. Finally, the lawyer would say that he is almost certain that the share of assets that constitutes “substantially all” is between 0.4 and 0.8.

These views mean that the relationship between the compliance effort $x$ (here, the percentage of assets transferred) and the probability of success $F(x)$ is

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75. See Sarah B. Lawsky, Probably? Understanding Tax Law’s Uncertainty, 157 U. PA. L. REV. 1017, 1022-23 (2009) (arguing that with a few exceptions, “legal scholarship has not focused on the question of what probability statements mean” and that adopting a particular meaning of probability statements “may reverse other legal scholars’ results”).

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not linear. The probability of success remains very close to zero for a range of low values of \( x \), and it is very close to 100% for a range of high values of \( x \). In some middle region, however, the probability of success increases relatively fast. In the current example, this region is between 50% and 70% of the assets (0.5 < \( x \) < 0.7). Thus, the relationship between the compliance effort \( x \) and the probability of success \( F(x) \) is S-shaped—it combines two areas of slow increase for very weak and very strong positions and an area of fast increase in the middle. This non-linear relationship is depicted as a dotted line in Figure 1.

The non-linearity assumption reflected in the S-shaped probability of success curve is not based on prior literature. Rather, it reflects my experience in interpreting and explaining legal standards as well as in giving legal advice. I believe, however, that the vast majority of lawyers will view this assumption as entirely uncontroversial in a very large number of cases. The vast majority of lawyers would also agree that, even though they often describe the likelihood of success to their clients in probabilistic terms, they do not interpret terms like “due” or “substantially all” by imagining a distribution of outcomes from multiple re-litigations of the client’s transaction. Instead, lawyers generally consider the relevant authorities and exercise their judgment. In the language of the probability theory, the probabilities that lawyers articulate are subjectivist, not frequentist; they reflect beliefs, not lotteries.

It is possible, however, to re-conceptualize the S-shaped curve in frequentist terms. This would allow us to deploy the basic analytical tools of the probability theory. Recall that lawyers believe that “substantially all” likely means 60% of the assets; it is highly likely to fall in the 50%-70% range; and it almost certainly lies in the 40%-80% range. These beliefs reflect legal uncertainty—the possibility that some legal experts would draw the line at some percentage of the assets other than sixty. We can envision a distribution of views about the value of \( x \) expressed by hypothetical experts (regulators,

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77. Experimental psychology has developed a very similar view of how people make uncertain judgments about probabilities. See, e.g., David Piercy, *Motivated Reasoning and Verbal vs. Numerical Probability Assessment: Evidence from an Accounting Context*, 108 ORGANIZATIONAL BEHAV. & HUM. DECISION PROCESSES 330, 331-32 (2009) (discussing the “membership function”). The discussion in the text applies to interpretations of all uncertain statements, not only probabilistic ones.

78. In fact, the members of the Tax Forum and the Tax Club (mostly tax partners at the leading New York firms) found the S-shaped curve to be self-evident. In the academic literature, Mark Gergen presents a related S-shaped curve. See Mark P. Gergen, *The Logic of Deterrence: Corporate Tax Shelters*, 55 TAX L. REV. 255, 282 (2002). Gergen’s curve is essentially identical to the one presented here, but only if we assume a constant relationship between the cost of taking a position (Gergen’s \( x \)-axis, and \( C(x) \) in the present model) and the compliance effort (the \( x \)-axis in Figure 1). This, however, would be a highly implausible assumption. In a different article, Gergen assumes a linear rather than an S-shaped relationship between the aggressiveness of a tax position (i.e. “probability tax is owed”) and the expected loss from an audit with automatic penalty. See Mark P. Gergen, *Uncertainty and Tax Enforcement: A Case for Moderate Fault-Based Penalties*, 64 TAX L. REV. 453, 485 (2011).

79. For an extended discussion of this point, see Lawsky, *supra* note 75. A frequentist probability is an expectation based on a large number of objective observations, such as multiple coin tosses. A subjectivist probability is a number reflecting the strength of a person’s belief.
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judges, or legal advisors\(^{80}\)) answering the question: “What fraction of the assets amounts to ‘substantially all’ if you had to give a single number?” Answers to this question would produce a distribution of views that may be described by a density function \(f(x)\). Assume that the answer given most frequently would be 60\%, producing the peak of the distribution. A large majority of experts would give an answer that falls between 50\% and 70\%, yielding high density near the peak. And almost everyone’s answer would be somewhere between forty and 80\% of the assets, meaning that the probability density outside of this range is very low.\(^{81}\) Finally, everyone would agree that transferring no assets means certain noncompliance while transferring all assets amounts to certain compliance. Thus, the density function is bounded on a [0,1] range.

We can assume that the density function \(f(x)\) is continuous and differentiable.\(^{82}\) This function has a corresponding cumulative distribution function \(F(x)\).\(^{83}\) Each value of \(F(x_0)\) is the probability that the actor would be allowed to claim the benefit \(b\) upon choosing the compliance effort \(x_0\) given the density function \(f(x)\).\(^{84}\) As long as the density function has a single peak—that is, as long as legal experts’ views converge toward a particular interpretation of an uncertain law—the distribution function has the S-shape identical to the intuitive probability of success curve reflecting subjectivist probabilities. In fact, the dotted line in Figure 1 reflects both the distribution function \(F(x)\) and the subjectivist probability curve described above. Therefore, the subjectivist and the frequentist understandings of legal uncertainty can be merged for the purposes of the model considered here.\(^{85}\) The following discussion does not distinguish between the two.

Note that both of these understandings differ from yet another meaning of uncertainty sometimes encountered in the literature. For instance, Petro Lisowsky and co-authors posit “a continuum from highly certain (least

\(^{80}\) Of course, the views of private lawyers and government rulemakers may diverge—a possibility considered infra Section II.E.

\(^{81}\) If these answers result from an actual large-numbers survey, the resulting probability density function is frequentist (with respect to experts’ views). If, more realistically, the answers are the product of a legal expert’s imagination, the resulting distribution is subjectivist, but with a frequentist flavor. Either way, the imaginary multiple answers allow us to construct a probability density function that an exercise of a legal judgment cannot produce.

\(^{82}\) This is a standard assumption. See, e.g., Dari-Mattiacci, supra note 3, at 171.

\(^{83}\) The density function \(f(x)\) is a derivative of the cumulative distribution function \(F(x)\). That is, \(F'(x) = f(x)\), \(F(x) = \int_0^x f(x) dx\), \(F(x_0) = 0\), and \(F(x_u) = 1\), where \(x_0\) and \(x_u\) are lower and upper bounds of the range on which \(F(x)\) is defined (for our purposes, this range is [0,1]).

\(^{84}\) As \(x\) increases from 0 to 1, so does \(F(x)\). In terms of our example, transferring no assets at all (\(x=0\)) is certain to flunk the “substantially all” test (\(F(x)=0\)), while transferring all assets (\(x=1\)) is certain to satisfy it (\(F(x)=1\)). Thus, the probability of success is non-negative and increasing in \(x\).

\(^{85}\) This conclusion also makes it clear that the term “uncertainty” used here, by C&C, and in much of the literature on the subject, refers to what the probability theory calls “risk” as opposed to “ambiguity.” For a discussion, see Eric Talley, On Uncertainty, Ambiguity, and Contractual Conditions, 34 Del. J. Corp. L. 755, 763-65 (2009).
aggressive) to highly uncertain (most aggressive)” tax positions. Kyle Logue discusses positions that are close to “tax evasion, where there is little substantive legal uncertainty (i.e., the positions will almost certainly be shot down if detected).” Both of these statements focus on the uncertainty of a particular position given ambiguous law, not on the extent to which the law itself is uncertain. The uncertainty of a position and the uncertainty of a legal standard are related but different. The former is endogenous while the latter is exogenous. A regulated party has no power to clarify the uncertain legal standard (change the shape of \( F(x) \)), but is free to eliminate all uncertainty by choosing certain compliance or something close to it. This Article’s focus is on the relationship between the exogenous uncertainty of legal standards and the endogenous choice of compliance efforts.

**D. The Basic Results**

Having specified the relationship between the compliance effort \( x \) and the probability of success \( F(x) \) as well as the meaning of the \( F(x) \) function, we can now get a sense of how the expected gain from taking an uncertain position varies with the compliance effort. We start with the most basic case and address multiple complications later. Figure 1 demonstrates some possibilities graphically for a simplified gain function featuring a linear cost and a benefit normalized to unity. Continuing with the example of a corporate transfer of “substantially all” of the assets, Actor 1 views asset transfers as less costly than Actor 2. The gain curves \( G_1 \) and \( G_2 \) reflecting the private gains of the two actors result from subtracting the respective cost curves from the same expected benefit curve represented by the dotted line. So, for example, when \( x=0.6 \), \( bF(x)=0.5 \), and \( C_1(x)=0.3 \). Subtracting the latter from the former (or, alternatively, adding 0.5 to -0.3) produces the value of 0.2 on the \( G_1 \) curve.

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86. Petro Lisowsky et al., Do Publicly Disclosed Tax Reserves Tell Us About Privately Disclosed Tax Shelter Activity?, 51 J. ACCT. RES. 583, 584 n.2 (2013).
88. In this model’s terms, there is no uncertainty if \( x=1 \) nor if \( x=0 \). This is Logue’s view. See Logue, supra note 87. Lisowsky et al. appear to suggest that most aggressive positions (where \( x \) approaches 0) are highly uncertain—a difficult conclusion to defend. See Lisowsky et al., supra note 86.
89. For instance, convex cost functions are considered later in this section, and in infra Sections II.A and II.B, infra. Non-differentiable cost functions are discussed in infra Section III.B. Other complications are introduced infra Part III as well.
90. This simplified function has the form of \( G(x) = F(x) - kx \). The linear cost function means that the company views a transfer of each additional unit of assets as equally undesirable. Normalization to unity (that is, assuming that \( b = 1 \)) is a standard move in economic modeling that eliminates a distraction of a non-consequential fixed variable. See, e.g., Christopher F. Baum et al., Securities Fraud and Corporate Board Turnover: New Evidence from Lawsuit Outcomes, 48 INT’L REV. L & ECON. 14, 19 (2016).
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Figure 1: The S-Shaped Probability of Success Curve and the Illustrative Cost and Gain Functions

Several observations follow readily from Figure 1. First, the gain function is wavy—it has a minimum and a maximum. This presents a problem for determining the actor’s privately optimal compliance effort $x^*$ algebraically. C&C faced the same problem and resolved it by relying on several assumptions. Two of these assumptions are less warranted beyond the types of settings (such as speeding and pollution) that served as their primary examples. The third one is restrictive and is not made here. Because it is

91. The probability of success curve $F(x)$ and the curve $bF(x)$ are the same because $b = 1$. $x_1^*$ and $x_2^*$ are the privately optimal compliance efforts of Actor 1 and Actor 2. $x_1^*$ yields the maximum gain of $G_1$ while $x_2^*$ yields the maximum gain of zero (meaning that Actor 2 will not engage in the transaction). $x_1^*$ corresponds to the local maximum $G_2^*$ on the gain curve for Actor 2. This maximum is local because that actor would rather abstain from acting than act and expect a negative gain ($G_2^* < 0$).

92. The private maximum corresponds to the first derivative $G'(x) = 0$ (this is the first-order condition or FOC) and the second derivative $G''(x) < 0$ (this is the second-order condition or SOC). In the general case, the SOC is $G''(x) = bF'(x) - C''(x)$. The sign of this expression may be positive or negative as each term may be greater or smaller than zero. In the simple case depicted in Figure 1, the SOC is $G''(x) = f'(x)$. If $f'(x)$ has a single peak, as the discussion assumes throughout, any $x$ satisfying the SOC that exceeds the peak corresponds to the maximum.

93. See Calfee & Craswell, supra note 1, at 967, 968, 970, 980.

94. C&C conclude that the private gain function has an internal maximum by assuming that the SOC is negative. See Craswell & Calfee, supra note 1, at 282 n.7. They explain that the first three terms in their SOC are unambiguously negative. However, the first term $B'(x)$ is negative only if the private benefit function is concave, as C&C assume on page 280. The second term is positive (it is being subtracted) only if $C''(x) > 0$ (the cost function is convex), as C&C assume on page 280 as well. (An alternative, less restrictive assumption that C&C also sometimes make is that $B'(x) < C'(x)$. This allows for a concave cost function and makes it less likely that the first three terms in the SOC are negative.) So the first three terms are indeed negative only as long as C&C’s assumptions hold. As
impossible to solve the model algebraically in the general case, the following analysis relies on simulations (much of C&C’s analysis does the same).

The second observation is that the privately optimal compliance effort \( x^* \) —the value of \( x \) corresponding to the highest point on the gain curve—may lie inside the [0,1] interval or at zero. That is, a rational actor facing an ambiguous legal standard may choose to take an uncertain position \( (x_1^* = 0.8 \text{ for Actor 1}) \) or abstain from acting altogether \( (x_2^* = 0 \text{ for Actor 2}) \). For a given likelihood of success (same \( F(x) \)), higher costs reduce the expected gain and may make the gamble not worth taking. None of this is surprising. C&C clearly showed that rational actors may take uncertain legal positions. They did not investigate the choice of not acting at all, but their model would easily produce this result.

mentioned in the text, these are plausible assumptions when one thinks about accident-producing behavior, but not in many other cases. For instance, the benefits from gradually monopolizing an industry, from increasing network coverage over a given territory, or from investing in technology whose adoption involves a tipping point are all likely to be convex rather than concave (as C&C assume) at least over some range of \( x \).

As C&C state (and assuming that the sum of the first three terms is indeed negative), the SOC is negative “as long as \( F'(x) L(x) \) never takes on a large negative value . . . If the density function is single-peaked [an assumption also made here], this is equivalent to assuming that it does not fall away extremely rapidly at values of \( x \) above the value at which it peaks. For the remainder of the paper, we will make this assumption.” Craswell & Calfee, supra note 1, at 282 n.7. The shape of the benefit and loss function, as well as the density function, affect what “extremely rapidly” means in any given case. The general point, however, is that as the law becomes increasingly clear—as uncertainty declines—the density function does indeed begin to fall extremely rapidly at values of \( x \) above the peak. At that point, C&C’s model no longer yields a clear prediction of individual behavior even given a concave benefit and convex loss functions that they assumed.

Technically, the problem arises because it is impossible to sign the SOC. By the “general case,” I mean the case where the cost function is not necessarily linear and the restrictive assumptions made by C&C may not be made.

I produced simulations using Microsoft Excel. I input the \( x \) values (from 0 to 100, or 0 to 1,000 to match C&C’s approach) manually as the first column. The second column reflected the corresponding values of a cumulative distribution function (CDF) \( F(x) \). Where possible (such as for a normal distribution), these values were generated using Excel’s built-in formulas and manually chosen CDF parameters (such as the mean and the standard deviation for a normal distribution). In the absence of built-in formulas (such as for a triangular distribution), the CDFs were constructed manually. The third column contained the values of a cost function corresponding to each value of \( x \). For these Article’s original simulations, the cost function was either linear or convex. For replicating C&C’s simulations the cost function matched the one that they used. The fourth column contained the values of the \( G(x) \) function reflected in supra note 48 for the Article’s original simulations and supra note 50 for the replications of C&C’s simulations. I used a built-in Excel function to find the maximum value of the gain function in the \( G(x) \) column. I then identified the values of \( x^* \), \( F(x^*) \), and \( G(x^*) \) produced by the distribution with particular parameters (such as a specific mean and standard deviation of a normal distribution). Finally, I repeated this process while varying the CDF parameters and CDF functional forms.

This result may be shown algebraically as well, and it is not restricted to linear cost functions. Assume that the cost function has a form of \( C(x) = k x^t \) (\( t = 1 \) corresponds to a linear cost, \( t = 2 \) to a linear marginal cost, and higher values of \( t \) correspond to steeper cost functions). Differentiating the gain function with respect to the two cost factors yields: \( \frac{\partial G(x)}{\partial k} = -x^t \), which is negative for any \( x > 0 \). \( \frac{\partial G(x)}{\partial t} = -k x^t \ln x \), which is non-positive for any \( x > 0 \) and \( t \geq 1 \).
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Another insight suggested by Figure 1 is that higher compliance costs induce rational actors to diminish their compliance efforts. For example, $x_1^*$ in Figure 1 is greater than $x_2^*$ (the effort corresponding to the local, rather than global, $G_2$ maximum). This is not an accident. Consider what would happen if the cost of compliance were to go down to zero. The gain curve would then coincide with the dotted S-shaped line, and the maximum gain would be reached for $x^*$ equal to one—the position assuring certain compliance. If compliance is costlier, however, perfection no longer makes sense.

These results are useful but hardly path-breaking. One reiterates C&C’s well-known finding; others follow from their model. Another question arising from examining Figure 1 is quite different. It is addressed next.

E. From Deterrence to Compliance

The simple yet fundamental question is this: How should we evaluate the actor’s compliance effort, $x^*$? In the optimal deterrence framework adopted by C&C, the answer is clear: we should evaluate $x^*$ by comparing it to the socially optimal effort $x^{so}$. The closer the $x^*$ is to $x^{so}$, the better. If $x^*$ exceeds $x^{so}$, we have under-deterrence; if $x^*$ is below $x^{so}$ then over-deterrence results. Think of 60 mph as the socially optimal speed and the uncertain negligence standard producing a single-peak density function centered on 0.6. If the standard induces drivers to go 90 mph, uncertainty leads to under-deterrence; if they drive 55 mph, the law slightly over-deters.

But what if we have no view about the efficiency of the law? How should we assess actor’s choices in the absence of the socially optimal level as a reference point? To the best of my knowledge, this deceptively simple question has not been addressed in the economic analysis of law.

This is a fundamental problem that must be resolved before the analysis can move forward. For instance, the value of $x_1^*$ in Figure 1 is 0.8. Is this low or high? How should we evaluate a particular position when the law is neither certain nor socially optimal?

There are at least two possible ways of doing so. First, we may ask what value of $x$ we would pick if we were choosing a single number as the best approximation for separating legal and illegal conduct. The most reasonable

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100. As mentioned earlier, Actor 2 would not choose $x_2^*$, but rather abstain from acting altogether.
101. In Shavell’s model, the answer is also clear: we should compare $x^*$ to the exogenous level of due care.
102. The only alternative to the optimal deterrence theory in the law and economics literature is the so-called complete deterrence approach. That approach does not assume socially optimal legal rules, but it generally treats legal rules as clear, giving rational actors a binary choice of complying or violating the law. In that setting, the obvious way to evaluate a private decision is by checking whether it is legal. For a discussion, see infra text accompanying notes 215-218. Importantly, in both the optimal and the complete deterrence settings, there is a precisely specified value of $x$ that separates desirable from undesirable behavior.
answer is the value that most legal experts would identify if they had to give a single number for the meaning of an uncertain standard. This is the value of \( x \) corresponding to the peak of the density function \( f(x) \)—the value reflecting the prevailing, most common interpretation. For many plausible probability distributions, this peak value corresponds to \( F(x)=0.5 \), the 50% probability of success.\(^{103}\) Let us call this peak value \( x^p \).

A useful feature of \( x^p \) is that, to put it bluntly, it is the only clear benchmark other than zero and one for a single-peak probability distribution.\(^{104}\) It reveals information in a setting where no other obvious benchmark exists. And this information is useful. Most importantly, focusing on \( x^p \) highlights the relativity of \( x^* \). It is impossible to decide whether any given \( x^* \) is high or low without knowing the corresponding \( x^p \).

This point is obvious to any legal practitioner. If a company must transfer “substantially all” assets to secure a regulatory benefit (a fairly high \( x^p \)), transferring 60% may not be enough. If, instead, the threshold is something like “material” share of assets (a much lower \( x^p \)), the same 60% level is almost unassailable.

Finally, \( x^p \) has a clear connection to Shavell’s analysis. He models uncertainty as an error term around a particular value of \( x \). He places very few restrictions on the shape of the error distribution, but his key assumption is that the magnitude of error declines as uncertainty declines.\(^{105}\) The most plausible interpretation of this assumption is that as a vague standard becomes clearer, the distribution of possible interpretations of that standard converges to \( x^p \).\(^{106}\)

The second way of evaluating \( x^* \) is by investigating the corresponding probability of success \( F(x^*) \). This approach shifts focus from deterrence to compliance, from the territory familiar to legal economists to the territory familiar to legal practitioners.

The importance of the probability of success is self-evident to any lawyer. It is especially familiar to any counselor engaged in opinion practice. Legal advisors have developed numerous opinion levels ranging from “will” (very high likelihood of success) to “reasonable basis” (low but not negligible

\(^{103}\) This is true for any symmetric distribution such as a normal distribution.

\(^{104}\) This is clearly true for symmetric distributions because for these distributions, the mean, the median, and the mode are the same. For skewed distributions, the mean and median must be calculated, and are sometimes not defined. In contrast, the mode (the technical term for \( x^p \)) is a defined value that may be produced by a legal expert’s introspection for any single-peak distribution. Note that the mode is not defined for a uniform distribution. But this distribution is not single-peaked. It is equivalent to legal experts thinking that any value of \( x \) is as likely to assure compliance as any other value of \( x \)—not a plausible description of reality.

\(^{105}\) See SHAVELL, supra note 1, at 94.

\(^{106}\) This is not true for a uniform distribution because it has no \( x^p \). However, as discussed in supra note 104, a uniform distribution is not a plausible depiction of legal uncertainty. The statement in the text may also not hold more generally in theory. But it is difficult to imagine, when talking about legal uncertainty in practice, how a single-peaked distribution of the views of legal experts would converge to some value of \( x \) other than \( x^p \) as uncertainty declines.
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likelihood of success) and even less plausible positions. To an outsider, these opinion levels are extraordinarily nuanced. For instance, the “will” level includes unqualified “will,” “not free from doubt will,” and “not entirely free from doubt will” opinions. Lawyers routinely suggest percentages corresponding to various opinion levels. There is some disagreement about what these numbers are, but everyone agrees that these percentages refer to the likelihood of success. And clients are extremely sensitive to these percentages and levels of “comfort.” Multi-billion dollar deals depend on lawyers’ conclusions regarding the likelihood that the transaction would withstand regulatory scrutiny. Thus, the realities of legal practice leave no doubt about the importance of $F(x)$.

The American Bar Association’s efforts to define the bounds of ethical legal advice further bolster the point. These efforts (some would say struggles) make it quite clear that the ABA views the likelihood of success as the key factor in separating ethical and unethical conduct. Equally clear is the ABA’s unease with positions that are highly unlikely to prevail.

Government regulators view the probability of success as an important factor as well. They also reveal a preference for more compliant (higher probability of success) positions. And they particularly disfavor positions that are more likely to be wrong than right.

The SEC, for instance, instructs issuers taking uncertain tax positions that fall short of the “will” level “to explain why [the counsel or accountant] cannot give a ‘will’ opinion.” The Commission does not even contemplate that issuers would go forward with transactions that are not supported by at least a “more likely than not” opinion of counsel. The FASB forbids the issuers from reflecting uncertain tax benefits in financial statements unless the issuer believes that there is more than a 50% chance that the benefit will be

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107. These are described by standards like “not frivolous” or a “reporting position.”


110. For example, the $33 billion Williams-Energy Transfer merger hinged entirely on whether one elite law firm reasonably refused to give a “should” opinion while another elite law firm believed that such an opinion would properly reflect the state of the law. See supra note 15.


112. See id. for a discussion of concerns with lawyers endorsing positions supported by merely a “colorable claim”—a less likely to succeed position than the one that has “some realistic possibility of success,” which, in turn, is less likely to succeed than a position supported by “substantial authority”—the threshold required by the Internal Revenue Code to avoid penalties for undisclosed positions. See I.R.C. § 6662 (2012).


114. Id.
sustained. Congress is willing to forgive tax penalties if “the taxpayer reasonably believed that [the] treatment was more likely than not the proper one.” And the Treasury Department likened legal opinions that fall short of the “more likely than not” level to advice plagued by conflicts of interest.

Overall, government regulators, self-regulatory bodies, and legal practitioners all care about the likelihood that an uncertain position would be upheld on review. So the second way of assessing the actor’s compliance effort is by evaluating the resulting probability of success.

Note that this probability has one obvious advantage over $x^p$. The latter is just one point in a distribution; the former is a continuous variable spanning certain compliance and certain noncompliance. Focusing on the probability of success leads to a greater precision in evaluating rational responses to legal standards even if the law does not maximize efficiency and the optimal deterrence theory is unhelpful. We now turn to this evaluation.

II. Investigating the Model

Adopting the probabilistic compliance framework allows us to analyze the model introduced in Part I—or the similar model studied by C&C—from a new perspective. We can investigate whether higher (or lower) legal uncertainty induces rational actors to take more compliant, higher probability of success positions. We can inquire into the private consequences of these decisions—with and without taking account of the market for legal advice. And we can ask how policymakers should react to private responses to legal uncertainty if policymakers care about compliance. This Part tackles these and related questions.

A. The Famous Ambiguity Reexamined

We begin this investigation by reexamining the best-known result reached by both C&C and Shavell regarding the effect of legal uncertainty on individual behavior—their conclusion that uncertainty may lead to either over- or under-deterrence even if the legal standard is set correctly on average. Shavell reaches this result with very few assumptions about the shape of legal uncertainty. However, his analysis is highly general, making it difficult to get a

118. For C&C, “correctly” means socially optimally. See Calfee & Craswell, supra note 1, at 974. For Shavell, “correctly” means at the level of due care that an actor would choose in the absence of uncertainty. See Shavell, supra note 1, at 97.
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sense of its practical implications. Moreover, no one has succeeded in extending Shavell’s model beyond his key finding in three decades. C&C obtained their result by relying on simulations, and this Article follows the same approach.

In order to generate their simulations, C&C needed to assume a functional form of legal uncertainty. As is common, they assumed that it is normally distributed, and this Article does the same. However, a normal distribution is defined on the range of plus/minus infinity rather than [0,1]. This Article’s original simulations truncate the distribution to fit the [0,1] interval by choosing such parameters (mean and variance) that the probability of occurrence (the density function) at either end of the [0,1] interval is equal to zero up to three decimal points. This assures that for \( x \) approximating zero (almost no effort to comply) the probability of success is almost zero as well, while for \( x \) approximating one (almost the highest possible compliance effort) the probability of success is also one (100%). As will become clear shortly, C&C did not impose similar restrictions in their simulations.

C&C interpret their results by focusing on what this Article calls the compliance effort \( x^* \). They determine privately optimal efforts corresponding to various degrees of uncertainty and compare them to the socially optimal value of \( x^{so} \). What would happen if we shift the analytical framework from deterrence to compliance, from comparing \( x^* \) to \( x^{so} \) to evaluating the value of \( F(x^*) \)?

It turns out that this question has the same answer if we consider this Article’s model or C&C’s formula. To emphasize the general nature of that answer, it is presented here using C&C’s original simulations. Table 1 replicates one of their scenarios (a combination of the gain and loss functions and a normal distribution with a particular mean) in the first two columns.

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119. Conceptually, of course, the general form of Shavell’s proof makes it especially valuable. But practically, his conclusion that uncertainty leads to over-deterrence as long as “the distribution of the deviations is not too dispersed” gives the reader no way of assessing what “too dispersed” might mean. In contrast, simulations offer a better sense of the extent of uncertainty, including in situations when the results become questionable. See infra text accompanying notes 130-133.

120. For a recent example of using a normal distribution assumption to model legal uncertainty, see Hoeppner & Lyhs, supra note 71, at 6. A related literature on litigation settlement predominantly relies on this assumption as well. See Marc Poitras & Ralph Frasca, A Unified Model of Settlement and Trial Expenditures: The Priest-Klein Model Extended, 31 INT’L REV. L. & ECON. 188, 190 (2011).

121. This is a crude solution. A transformation that would convert the distribution’s \([-\infty, +\infty]\) range into [0,1] range would be preferable. This was not the path followed by C&C, however, and I do not follow it here.

122. In other words, this assures that \( F(x) \) satisfies one of the basic features of a cumulative density function. See supra note 83.

123. See Craswell & Calfee, supra note 1, at 284 tbl.1 (column corresponding to the mean of 500).
The third column is new, and it shows the values of the probability of success for each choice of $x^*$ reported by C&C.\textsuperscript{124} The results are quite striking.\textsuperscript{125}

Table 1: The Effect of a Change in Uncertainty on the Privately Optimal Compliance Effort and the Corresponding Probability of Success

<table>
<thead>
<tr>
<th>St Dev</th>
<th>$x^*$</th>
<th>$F(x^*)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>497</td>
<td>1.00</td>
</tr>
<tr>
<td>10</td>
<td>476</td>
<td>0.99</td>
</tr>
<tr>
<td>50</td>
<td>425</td>
<td>0.93</td>
</tr>
<tr>
<td>100</td>
<td>407</td>
<td>0.82</td>
</tr>
<tr>
<td>200</td>
<td>429</td>
<td>0.64</td>
</tr>
<tr>
<td>300</td>
<td>466</td>
<td>0.55</td>
</tr>
<tr>
<td>400</td>
<td>500</td>
<td>0.50</td>
</tr>
<tr>
<td>500</td>
<td>529</td>
<td>0.48</td>
</tr>
<tr>
<td>1000</td>
<td>626</td>
<td>0.45</td>
</tr>
</tbody>
</table>

The first two columns show that even when legal uncertainty is symmetric and centered on the social optimum (here, $x^*=x^p=500$) both under- and over-deterrence are possible. The behavior is optimal when uncertainty is vanishingly small (the distribution’s standard deviation $\sigma$—a basic statistical measure of uncertainty—is equal to 1) or very significant ($\sigma=400$).\textsuperscript{126} But when uncertainty is high, under-deterrence results,\textsuperscript{127} and when uncertainty is lower we observe over-deterrence. Moreover, the relationship between uncertainty and deterrence is non-monotone. Starting with extreme uncertainty ($\sigma=1000$), a decrease in uncertainty first moves the actor’s choice from under-deterrence to the optimum ($1000>\sigma>400$), then produces increasing over-deterrence ($400>\sigma>100$), and finally reduces over-deterrence bringing the behavior back to the optimum ($100>\sigma>1$). In stark contrast, when we turn from deterrence to compliance (that is, the probability of success $F(x)$), the relationship is much more straightforward: lower uncertainty leads to greater compliance.

This is no small difference. Private choices viewed as equivalent in C&C’s framework are anything but equivalent in the present one. For instance, C&C’s simulations show that both high uncertainty ($\sigma=400$) and extreme

\textsuperscript{124} Because C&C use $F(x)$ as a probability of liability (bad outcome), while I use $F(x)$ as a probability of success (good outcome), the $F(x)$ values reported in the table correspond to $(1-F(x))$ values in the C&C’s simulations.

\textsuperscript{125} All simulations and graphs are available upon request to the author.

\textsuperscript{126} This and similar statements treat $x^*=497$ as essentially equal to 500.

\textsuperscript{127} Recall that in C&C’s setup, higher values of $x^*$ are less socially desirable (think of higher speeds), while in the present setup higher $x^*$ reflects greater compliance (think of transferring more assets to satisfy the “substantially all” standard).
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certainty ($\sigma=1$) induce private actors to take the socially optimal position ($x^*=500$). But from the compliance perspective, the two could not be more different. High uncertainty ($\sigma=400$) leads merely to a plausible but hardly indisputable fifty-fifty position ($F(x^*)=0.5$), while extreme certainty induces certain compliance ($F(x^*)=1$). Relatedly, some changes viewed as undesirable in C&C’s optimal deterrence framework are clearly desirable if one prefers more compliance to less. For instance, as the standard deviation changes from 400 to 100 (legal uncertainty decreases), the behavior gets worse ($x^*$ falling away from $x^*$) from the optimal deterrence perspective while those concerned with compliance would conclude that the behavior improves ($F(x^*)$ increases from 0.48 to 0.82).

These results are not idiosyncratic. In fact, every C&C scenario featuring a symmetric distribution centered on $x^*$ yields the same monotone relationship between uncertainty and the likelihood of success. Moreover, the same is true if we ignore $x^*$ and focus on $x^*$, similar to Shavell. In each of the forty scenarios reflecting symmetric distributions the same result holds: lower uncertainty leads to greater compliance. Notably, this is true regardless of the location of the distribution’s mean in the [0,1] range.

There are two important caveats to this strong claim. First, sometimes a change in uncertainty may not matter. If, for example, an actor chooses to abstain from taking an uncertain position, a change in uncertainty may make the actor even less likely to act. Second, in some of C&C’s scenarios (nine out of forty), the monotone relationship between uncertainty and compliance does not hold for the entire range of uncertainty they study. However, in each case where this relationship reverses, a normal distribution is not a plausible representation of legal uncertainty. In those cases, the level of uncertainty producing the reversal is so high that the lowest and/or highest values of $F(x)$ are not close to zero or one. This means that even outright evasion leads to a significant likelihood of success (recall that perfect detection is assumed), and even perfect compliance results in a significant probability of being viewed as illegal. In contrast, both the frequentist and the subjectivist understandings of legal uncertainty demand that the relevant probability varies from zero to one within the uncertain range. Thus, we should take C&C’s scenarios incorporating the $F(x)$ curve that does not start close to zero and/or does not

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128. These are C&C’s Tables 1-5 with eight columns per table. See Calfee & Craswell, supra note 1. By symmetric, I mean non-skewed.

129. Furthermore, extending C&C’s simulations to distributions with means closer to the ends of the [0,1] range yields scenarios where the change in uncertainty does not affect the location of $x^*$ even when actors choose to take uncertain positions. In those additional simulations, $x^*$ is determined wholly by the cost and benefit functions that C&C chose for their examples.

130. Rather, $F(x)$ corresponding to no effort to comply ranges from 8% to over 20%, increasing to over 30% as the standard deviation reaches 1,000.

131. Relatedly, an $F(x)$ function that does not start at 0 and end at 1 fails to satisfy the basic requirements of a cumulative distribution function. See supra note 83.
end close to one with great caution. In all instances where this problem does not arise, lower uncertainty leads to greater compliance in C&C’s specifications.

One may wonder if this outcome is due to some peculiarity of a normal distribution. It is not. Re-running one of C&C’s scenarios using three plausible alternative distributions—Logistic, Cauchy, and Triangular—makes no difference. All these simulations produce the results similar to those shown in Table 1: the relationship between uncertainty and $x^*$ is sometimes non-monotone but lower uncertainty always leads to greater compliance.

The model offered in this Article yields the same relationship as well. Figure 2 offers a graphical illustration. Panel A presents three increasingly certain density functions (the tighter the bell curve, the more certain is the function) and a linear marginal cost line. Following Table 1, all distributions are centered on 0.5. The privately optimal values of $x^*$ correspond to intercepts between the marginal cost line and each density function that lies to the right of that function’s peak. These intercepts are marked as low, medium, and high, reflecting the levels of uncertainty. The accompanying numbers in callout boxes are the precise values of the respective $x^*$.

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132. This is an important caveat because contemporary economic research building on C&C’s analysis continues to study legal uncertainty using distributions with excessively high degrees of variance. See Hoeppner & Lyhs, supra note 71.

133. I was unable to replicate C&C’s results for a skewed (not normal) distribution, see Craswell & Calfee, supra note 1, at 291 tbl.6, because I could not guess the three parameters underlying the distribution.

134. See Craswell & Calfee, supra note 1, at 288 tbl.3 (the column with the mean value of 500). I chose Table 3 for no particular reason, other than that the benefit and loss functions modeled in Table 3 are appealingly simple and, therefore, transparent.

135. These distributions are plausible because (i) they are continuous, (ii) they are single-peaked, (iii) they may have a mean other than zero (Student’s T distribution, for instance, always has a zero mean), (iv) they produce (in their pure or truncated form) S-shaped $F(x)$ functions on the [0,1] range for various levels of uncertainty, and (v) they are defined by their parameters in such a way that varying one of the parameters changes the dispersion of the distribution without changing its mean (something that is not possible, for instance, for Beta, Gamma, Chi, and some other distributions).

136. Panel A is a graphical illustration of the first-order condition for the gain function. The second order condition is $G''(x) = f'(x)$, and it is negative (corresponds to a maximum $G(x)$) where $f(x)$ is declining (i.e., to the right of the peak).
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Figure 2: The Effect of a Change in Uncertainty on the Probability of Success Function

As is clear from Panel A, a decrease in uncertainty from high to medium moves $x^*$ to the right (from 0.58 to 0.6), while a further decrease from medium to low moves $x^*$ to the left (from 0.6 to 0.55). This reflects the complicated relationship between uncertainty and $x^*$ revealed in C&C’s simulations. Panel B tells a different story. It shows the familiar S-shaped curves, with steeper lines corresponding to tighter, less uncertain probability distributions. Again, a decrease in uncertainty shifts $x^*$ first to the right and then to the left, reflecting the shifts depicted in Panel A. But in contrast with Panel A, each shift leads to an increase in the optimal probability of success $F(x^*)$. The numbers in the low/med/high boxes restate the respective optimal compliance efforts $x^*$ and add the corresponding values of $F(x^*)$. Clearly, the probability of success increases monotonically as uncertainty declines.

The complex relationship between uncertainty and the compliance effort is not inevitable. One can easily produce simulations where a decrease in uncertainty leads to smaller values of $x^*$ (always moves $x^*$ to the left) and higher values of $F(x^*)$. The important point, however, is that whatever the relationship is between uncertainty and $x^*$, lower uncertainty appears to lead to more conservative, higher probability of success positions. For those interested in legal compliance, this is a significant finding. A natural question to ask next is what this means for private actors.

B. Legal Uncertainty and Private Gains

Rational, risk-neutral actors care about their gains. How do these gains change when a legal standard becomes more rule-like? Figure 3 suggests the answer. It depicts the same S-shaped curves that appear in Figure 2, adds a linear marginal cost curve (the downward sloping dashed line reflecting the
cost function depicted in Figure 2, Panel A\(^{137}\), and presents the gain curves resulting from subtracting that cost from each of the three expected benefits \(bF_i(x)\).

The relationship between private gains and uncertainty is not hard to see. Even in this case where the optimal compliance effort \(x^*\) changes non-monotonically as uncertainty decreases, greater certainty leads to greater gains.

**Figure 3: The Effect of a Change in Uncertainty on the Gain Function**

This is not an obvious result when the relationship between \(x^*\) and uncertainty is not monotone. As just discussed, \(x_2^*\) is greater than \(x_1^*\). This means that \(C(x_2^*)\) is higher than \(C(x_1^*)\). Nevertheless, the benefit of greater certainty (a steeper \(F_2\) compared to \(F_1\)) exceeds the additional cost of moving from \(x_1^*\) to \(x_2^*\), resulting in a higher peak of the \(G_2\) curve compared to \(G_1\). When the relationship between the compliance effort and the uncertainty level is monotone, the same result is obvious. Lower uncertainty leads to both less costly compliance efforts and higher expected benefits due to steeper S-shaped curves. Both effects increase the maximum gain.

C&C’s simulations produce similar results. Out of forty scenarios that they study, only two\(^ {138} \) reveal a non-monotone relationship between gains and uncertainties.

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\(^{137}\) The cost curve in Figure 3 is curved, while it is straight in Figure 2, Panel A, because the curves in Figure 2, Panel A (both the cost curve and the probability curves) reflect functions that are first derivatives of the functions that produce the cost and the S-shaped probability curves in Figure 3.

\(^{138}\)
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uncertainty for the levels of uncertainty where a normal distribution is appropriate. An almost complete match in the relationship between gains and uncertainty in C&C’s simulations and the ones original to this Article suggests that this relationship is fairly robust.

The strong (even if not universal) correspondence between uncertainty and private gains may explain a phenomenon widely observed in legal practice: lawyers and their clients often complain about legal uncertainty. Even sophisticated actors such as large firms and their representatives lobby constantly to clarify, specify, and narrow the rules. Whether in tax, environmental regulation, or financial regulation, sophisticated actors tend to prefer rules to standards; that is, they want greater certainty. The model explains why this preference is rational even for risk-neutral parties: greater certainty likely means larger private gains.

C. Legal Uncertainty from the Government’s Perspective

What about the government’s perspective? Should the government prefer more certainty or less? To answer this question, we need to specify the government’s objective. The dominant economic view of what this objective ought to be is well-known—it is social welfare maximization. C&C’s analysis adopts this view, and Shavell’s study of uncertainty does the same implicitly.

This Article, in contrast, seeks to analyze many existing legal standards that may not be plausibly assumed to be welfare maximizing even approximately. The optimal deterrence theory is not particularly helpful or

138. See Calfee & Craswell, supra note 1, at 288 tbl.3 (the columns with the mean value of 350 and 400).

139. See, e.g., Kristen A. Parillo, Businesses Would Pay More Taxes for More Certainty, Survey Says, 148 TAX NOTES 1069, 1069 (2015) (summarizing a Grant Thornton survey of 2,580 businesses in 35 jurisdictions revealing that “three-quarters of companies would support paying more taxes in exchange for greater clarity from tax authorities regarding what constitutes acceptable tax planning”); Susan P. Serota, Economic Substance Codification: ABA Has “Substantial Reservations,” 115 TAX NOTES 389, 392 (2007) (reporting comments from the ABA Tax Section urging Congress to eliminate a provision in the proposed legislation that would create “an entirely new and uncertain requirement on a wide range of transactions”); Lee A. Sheppard, Partnership Antiabuse Rule Produces Anticlimactic Hearing, 64 TAX NOTES 558 (1994) (summarizing sharp disagreements, including within the tax bar, about the cost of uncertainty created by the proposed antiabuse regulations).


relevant for policymakers evaluating these real-life, imperfect, uncertain standards. Two alternatives suggest themselves.

The first one comes from the law and economics of contracts. The conventional assumption in that literature is that the overall efficiency may be approximated by focusing on private gains of the contracting parties.\textsuperscript{142} The same approach has been used in corporate governance scholarship. There, the social optimum is assumed to coincide with the maximum joint gain of shareholders and managers.\textsuperscript{143} Equating overall efficiency with private gains is plausible if externalities and other market failures are unlikely, as may be true in some settings. It is less plausible for government regulation generally. At the same time, if the law is inefficient, externalities themselves may not be welfare-reducing.\textsuperscript{144} In any case, in the absence of welfare-maximizing rules, private gains offer policymakers a useful evaluative criterion.

Another plausible goal that government actors may pursue is compliance. As already discussed, regulators clearly care about it; they prefer more compliance to less; they tend to disfavor positions that are more likely to be wrong than right; and they particularly dislike positions that have a very small chance of success.\textsuperscript{145} All of this seems to suggest that if maximizing social welfare is not a realistic objective, maximizing compliance is an appealing alternative. It seems simple, intuitive, and more reflective of the real world than either welfare or private gain maximization.

A moment’s reflection reveals, however, that compliance maximization is not what regulators want. Even though they generally prefer greater compliance, they do not require perfection. For instance, Congress could have required a transfer of “all” assets, rather than “substantially all,”\textsuperscript{146} in order to capture a tax benefit. Or it could have designated a fixed percentage of assets and penalized any company that failed to comply fully. The Supreme Court could have demanded that a governmental unit explains its permitting decisions “contemporaneously,” rather than “essentially contemporaneously.”\textsuperscript{147} Or the Court could have specified a precise number of days as a condition of sustaining the permitting decision. Why do regulators enact vague standards that all but invite less-than-full compliance, and then object when compliance is not close to perfect?

\textsuperscript{142} See, e.g., Ian Ayres & Eric Talley, Solomonic Bargaining: Dividing A Legal Entitlement To Facilitate Coasean Trade, 104 YALE L.J. 1027, 1082 (1995) (referring to “‘efficiency-minded’ lawmakers [who] might pursue the narrow goal of maximizing gains from trade”); Benjamin E. Hermalin, Avery Katz & Richard Craswell, Contract Law, in 1 HANDBOOK OF LAW AND ECONOMICS 1, 13 (A. Mitchell Polinsky & Steven Shavell eds., 2007).


\textsuperscript{144} See supra note 30.

\textsuperscript{145} See supra text accompanying notes 107-117.

\textsuperscript{146} I.R.C. § 368(a)(1)(C) (2012).

\textsuperscript{147} T-Mobile S., LLC v. City of Roswell, 135 S. Ct. 808, 812 (2015).
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This Article does not develop a theory of enforcement of uncertain legal commands. The rules-versus-standards literature that explains why both types of commands exist is well known. Its basic idea is that standards are preferable when the policymaker is uncertain where to draw the line and it is costly to resolve this uncertainty ex ante. When this happens, ex post case-by-case interpretations both assure appropriate results in individual cases and gradually convert a vague standard into a clear rule. Notably, if we think of the latter process as a convergence of a distribution toward the mean, a regulator should be indifferent about whether any particular interpretation exceeds or falls short of that mean. After all, a standard will become a rule over time no matter where individual interpretations happen to fall along the way.

Yet this indifferent attitude is clearly not what lawmakers reveal when they insist on “more likely than not” or stronger positions even when these positions are observed by the regulators, or when they impose penalties on positions with particularly low chances of success. So while the rules-versus-standards framework explains why lawmakers enact standards in the first place, it does not explain lawmakers’ observed preference for more compliant positions after the standard is put in place.

One possible explanation reflects multiple levels of regulatory authority. Congress enacted the “substantially all” test, but the IRS needs to administer it. For obvious reasons, courts would not allow the IRS to interpret “substantially all” as “all” (or as “nothing,” for that matter), so the agency must accept compliance efforts lower than $x=1$ (and higher than $x=0$) even if it prefers these efforts to be as high (low) as possible. This explanation, however, does not illuminate the preference for high but not certain compliance of Congress itself. Perhaps it is related to enforcement costs, or political economy considerations, or confusion between welfare-maximizing laws and those that are not, or some other reason. A model explaining this preference would be a great contribution to the literature. Unfortunately, it does not appear to exist. Thus, this Article’s compliance-focused analysis takes the regulators’ preferences as they are. Regulators clearly care about the probability of success; they often prefer this probability to exceed 50%, but they do not demand perfect certainty.

To be clear, this discussion does not establish that more compliant positions are better for the society than less compliant ones. Nor does it demonstrate that any particular probability of success is socially optimal, most fair, or otherwise preferable.

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149. See id. at 569.
150. See supra text accompanying notes 113-117.
151. See, e.g., I.R.C. § 6662(d)(2)(B)(ii) (2012) (imposing a penalty on positions found to be incorrect by a court if the positions are not supported by a “reasonable basis”—a threshold significantly below the fifty-fifty level of confidence).
But the discussion does suggest a way of evaluating the consequences of legal uncertainty from the government perspective. We can do so by asking whether, given regulators’ revealed preferences, some of the government’s views and enforcement strategies are inconsistent with the government’s compliance-oriented objectives. More specifically, we can identify mistakes in the regulators’ understanding of the relationship between legal uncertainty and probabilistic compliance. We can also highlight the consequences of government’s action (or inaction) following the enactment of vague standards. The following two sections take on these tasks in turn.

D. Government’s Confusion About the In Terrorem Effect

Uncertain legal commands are pervasive, their promulgators and enforcers heterogeneous. So one should be cautious with generalizations. It is fair to say, however, that many regulators believe that legal uncertainty has a desirable \textit{in terrorem} effect. Regulated parties, this view suggests, are reluctant to take aggressive, low likelihood of success positions if they are unsure of what is legal and what is not. Greater legal certainty, in contrast, emboldens private actors to be more aggressive.\footnote{Anecdotal support for this view is not difficult to find. See, e.g., Marie Sapirie, \textit{The Evolution of Inversions}, 148 TAX NOTES 611, 613 (2015) (citing a corporate tax expert’s view that “[w]hen you have the [standard] that has a lot of gray areas, you can’t risk being right at the line”).}

Government agencies have expressed this view.\footnote{See, e.g., Richard M. Lipton, \textit{Tax Administration in the 90s: The New “Reign of Terror”}, 74 TAXES 227, 234 (1996) (stating that “[t]here have been a number of instances in which the IRS has indicated its intention to use \textit{in terrorem} rules as the basis for tax regulation,” such as in the partnership area).} Legal advisors and commentators have stated it.\footnote{See \textit{Marvin Chirelstein, Learned Hand’s Contribution to the Law of Tax Avoidance}, 77 YALE L.J. 440, 459 (1968) (referring to the \textit{in terrorem} effect of uncertain rules); Michael S. Kirsch, \textit{Alternative Sanctions and the Federal Tax Law: Symbols, Shaming, and Social Norm Management as a Substitute for Effective Tax Policy}, 89 IOWA L. REV. 863, 900-01 (2004) (“The uncertainty created by the vague language of the provision and the lack of guidance generates an \textit{in terrorem} chilling effect on U.S. citizens considering expatriation.”); Martin J. McMahon, \textit{Reflections on the Regulations Process: “Do the Regulations Have To Be Complex” or “Is Hyperlexis the Manna of the Tax Bar?”}, 51 TAX Notes 1441, 1446 (1991) (“Finally, where the object of a particular provision is to protect the fisc against abusive transactions, consideration should be given to the potential \textit{in terrorem} value of vague general rules, even though they produce planning complexity.”) (emphasis added).} One way a regulator may exacerbate the \textit{in terrorem} effect is by withholding authoritative guidance that would clarify vague standards. The

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IRS, the Food and Drug Administration, and even State Bar Associations have all relied on this strategy. Another in terrorem tactic is to issue warnings about possible future punitive guidance.

The model offered here explains the likely intuition underlying the in terrorem view of legal uncertainty. It also demonstrates why this intuition is wrong.

The origin of this intuition is obvious from Figure 2. A decrease in uncertainty from moderate to low induces a downward shift in $x^*$ from 0.6 to 0.55. Such shifts seem undesirable to regulators. The lower the compliance effort, their intuition suggests, the lower the probability of success. The fewer assets the company transfers under the “substantially all” test, the more aggressive is the transaction.

This link between the compliance effort $x^*$ and the probability of success $F(x^*)$ is understandable, but mistaken. The link is understandable because it does indeed exist if the level of uncertainty is constant. But if uncertainty changes, the analysis changes as well. When the downward shift in $x^*$ results from a change in uncertainty, we cannot assume that a lower effort corresponds to a lower likelihood of success. Rather, we need to investigate how a change in uncertainty affects the probability of success directly. And as we now know from Section II.A, this probability generally increases with greater certainty even if the compliance effort declines.

The case should not be overstated. The results just discussed come from simulations. They hold if actors take uncertain positions in excess of fifty-fifty.

156. See, e.g., Kimberly S. Blanchard, Guidance Needed for CFC Lending Transactions, 126 TAX NOTES 201, 202 (2010) (describing how the IRS’s refusal to issue guidance has an in terrorem effect on taxpayers and their counsel); Kirsch, supra note 154 (mentioning the lack of guidance).


158. See, e.g., Deborah Rhode, Moral Character as a Professional Credential, 94 YALE L.J. 491, 517-18 (1985) (explaining that the lack of published guidelines for “character and fitness” investigations of applicants to State Bars, combined with the absence of advance rulings by character committees, leads to great uncertainty and many negative consequences for would-be lawyers).

159. See, e.g., Kimberly S. Blanchard, Extensive New Anti-Inversion Rules Issued, 145 TAX NOTES 89, 91-92 (2014) (referring to the in terrorem effect of the IRS warning that if the Service “can figure out that it has colorable authority to write [interest-stripping] rules,” they will do so in the future to deter inversions); Kirk Van Brunt, Tax Aspects of REMIC Residual Interests, 2 FLA. TAX REV. 149, 245 (1994) (arguing that the IRS’s “ominous rumblings . . . created a fair amount of confusion and uncertainty in the marketplace . . . and no doubt this in terrorem effect was intended”) (emphasis added).

160. As we have seen, a decline in uncertainty (say a shift from $F_1$ to $F_2$ to $F_3$ in Figure 2) may lead to a decline in the compliance effort $x^*$. Even when $x^*$ increases (as it does from $x_1^*$ to $x_2^*$), that increase is unstable. Eventually, greater certainty leads to a lower compliance effort, assuming the original effort exceeded $x^*$. For example, privately optimal compliance efforts in Figures 2 and 3 range from 0.55 to 0.6. Imagine that the law becomes perfectly certain, and the line separating legal and illegal conduct is precisely 0.5 (the mean of uncertain distributions in Figures 2 and 3). Whether an actor started at 0.55, 0.58, or 0.6, the actor will now choose the effort just above 0.5—he will reduce his compliance effort.
Even so, the results are useful. The government may often observe whether positions exceed the fifty-fifty threshold, and they are likely to do so in any event.\textsuperscript{161} And although simulations do not supply irrefutable proof that lower uncertainty leads to greater compliance, they show that very often it does. Moreover, the simulations demonstrate convincingly that a reduction of compliance efforts in response to lower uncertainty does not necessarily mean more aggressive, less compliant behavior. Thus, higher compliance efforts in response to greater \textit{in terrorem} uncertainty are nothing for the government to celebrate.

This discussion sheds further light on the two evaluative criteria suggested above: $x^p$ and $F(x)$. It must now be clear that the latter is not just superior, but is far superior to the former. $x^p$ is useful as a rough indicator of whether any particular $x^*$ is more likely to be legal or illegal. But $x^p$ is a poor guide otherwise. Assessing private actors’ compliance efforts by reference to $x^p$ may seriously mislead policymakers because it restricts their attention to compliance efforts. Rather, only by considering the values of $F(x)$ can regulators appraise the effect of a change in uncertainty—whether on private gains or on probabilistic compliance. The model demonstrates that if policymakers care about either, greater certainty should often be the government’s goal.

Thus, the DOJ and the FTC were probably wise to develop a rigorous methodology for defining relevant product markets rather than relying on a vague “effective competition” standard.\textsuperscript{162} In contrast, the IRS was likely ill-advised to replace a relatively clear definition of “substantial business activities” in its first set of anti-inversion regulations with a vaguer test. In contrast, the tax agency enhanced taxpayer compliance when it replaced that test with a clear bright-line rule.\textsuperscript{163}

More generally, if a regulator observes private actors’ responses to a vague standard, and if a less uncertain standard would satisfy the regulator’s preferences, the regulator should clarify the standard to increase compliance. Of course, if a regulator is unsure how to narrow a standard, the regulator should keep the law vague. In other words, the argument here is only against uncertainty for uncertainty’s sake.

\textbf{E. Legal Uncertainty and the Market for Legal Advice}

There exists yet another reason for the government to value legal certainty. That reason emerges when we consider the market for legal advice. This market involves sophisticated parties and privileged communications. Thus, the following discussion is somewhat speculative and is based on personal experience in private practice and conversations with numerous legal

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\textsuperscript{161} As discussed in Section II.F, \textit{infra}, this is the most likely outcome when a regulator observes the choice of $x$.
\textsuperscript{162} See supra note 35.
\textsuperscript{163} See supra text accompanying notes 7-11.
\end{flushleft}
experts. Those familiar with this market may decide for themselves whether the concerns described below ring true.

Until now, we have implicitly assumed that any change in uncertainty is due to government action. An administrative agency may issue guidance interpreting a vague standard, or a court may narrow the range of the standard’s possible meanings. However, the law may appear to become more certain to regulated parties for a very different reason. As legal advisors grapple with ambiguous terms, and as they interact with each other, they may (and often do) become increasingly confident in their interpretations of uncertain legal provisions even in the absence of any authoritative guidance. The perceived uncertainty declines without any change in the content of the law. Numerous “rules of thumb” that have emerged in securities regulation, tax, and antitrust enforcement, among others, are all examples of this process.

This perceived decline is likely to induce rational actors to shift toward the smaller values of $x^*$ as discussed in Section II.A. Notably, this will happen even if legal advisors interpret the vague standard correctly on average, that is, if the mean of the single-peak density function reflecting legal uncertainty remains the same. The perceived tightening of this distribution alone will often lead to lower compliance efforts.

In terms of the model, the $F(x)$ distribution will remain unchanged while the compliance effort $x^*$ will decline. This will yield weaker, more aggressive positions. Moreover, because greater certainty tends to lead to higher private gains, an increase in perceived certainty will induce some actors who previously abstained from acting altogether to go forward with uncertain transactions. Because the actual uncertainty has not diminished, these transactions are unlikely to reflect high probability of success. None of this would make law enforcers happy.

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166. For instance, the “small but significant and nontransitory increase in price” test established by the Horizontal Merger Guidelines is generally interpreted to mean “5-10 percent price increase lasting one year or longer.” Wegener, supra note 35, at 156.

167. A shift to higher values of $x^*$ is also possible, though unstable. See supra note 160.

168. Consider an actor, call him $X$, who is very similar to the actor described in Figure 3, except that $X$’s compliance cost is slightly higher than those of that actor. As a result, the tip of the $G_1$ curve for $X$ is just below zero and he abstains from acting. If $X$ perceives that uncertainty has decreased to a point where it is reflected by the steep curve $F_3(x)$, $X$’s gain becomes positive (similar to the top of the $G_1$ curve) and he takes an uncertain position with compliance effort similar to $x^* = 0.55$. If $F_3(x)$ was the true level of uncertainty, that compliance effort would lead to a position that is 98% likely to be correct. However, because the $F_3(x)$ remains the relevant curve, the same compliance effort corresponds to a much lower 63% probability of success.
Another common trend among legal advisors should make regulators even more concerned: these advisors are likely to view an uncertain legal standard as more and more forgiving. To see why, consider first what happens if the government gradually relaxes a legal standard. In terms of the model, the spread of the distribution (the steepness of the $F(x)$ function) stays the same, but $x^p$ shifts downward. The obvious consequence of this shift is a shift of $x^*$ in the same direction: A less demanding legal standard would lead rational actors to reduce their compliance effort. A slightly less obvious implication is that a decrease in $x^p$ leads to greater gains. This may induce some actors who were unwilling to engage in transactions facing a costlier standard (higher $x^p$) to go forward with the deal if the standard is easier to satisfy. Figure 4 reflects these considerations, holding all other parameters constant.\footnote{\textsuperscript{169} Specifically, as $x^p$ declines from 0.7 to 0.3, the peak of the gain curve also shifts to the right (that is, $x^*$ declines). In addition, maximum gains are higher for lower values of $x^p$.}

Figure 4: The Effect of a Change in the Legal Standard on the Gain Function

The trend depicted in Figure 4 is unobjectionable as long as the reduction in $x^p$ is due to new information from the authoritative interpreter of the law. However, a decrease in $x^p$ may have a very different explanation.

When legal advisors face a new uncertain provision, they adopt a particular interpretation and proceed with advice. Because any interpretation other than $x^p=0$ forces clients to incur a cost, clients pressure their advisors to
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interpret the standard in the least costly way. If regulators remain silent, this pressure produces a one-way ratchet.

For instance, lawyers begin by issuing “should” opinions stating that 80% satisfies the “substantially all” test. Some transactions go forward based on these opinions. If the regulator does not challenge these transactions, legal advisors assume that 80% satisfies the standard for sure. They begin to issue stronger “will” opinions to clients willing to transfer 80% of the assets, and “should” opinions to clients transferring less than that. Thus, the prevailing interpretation of “substantially all” shifts toward less costly positions.

In the model’s terms, this means a shift in the perceived distribution toward the lower values of \( x^s \), with a corresponding shift in the values of \( x^* \). But the actual distribution remains the same because the law has not changed since the enactment of the standard. Thus, a downward shift in \( x^* \) produces much more aggressive positions. Using Figure 4 as an example, a shift from the actual legal standard reflected (let us assume) by the probability of success curve \( F_2 \), to a perceived standard reflected by the curve \( F_3 \), reduces the probability of success of the client’s position from 91% to 19%. Generally, more actors take uncertain positions, and these positions become more aggressive. As with a decrease in the perceived uncertainty, this perceived relaxation of a legal standard undermines compliance.

The remedy for both problems is the same: the government should either clarify uncertain provisions or counter the false perceptions—be it a perception of an increased certainty or of increasingly lenient standards—that are likely to result from the market for legal advice.

Examples of regulators using all these approaches are not hard to find. The IRS officials, for instance, occasionally remind practitioners that the government does not recognize the so-called “Wall Street Rule.” Similarly, the SEC made a point of reiterating that it does not subscribe to the “rule of thumb” definition of “materiality” for the purposes of financial misstatements. And even though the Horizontal Merger Guidelines implicitly

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170. They do so rather than assuming, for example, that the resource-constrained regulator has not had a chance to consider the issue.

171. That is, \( x_3^* = 0.43 \), corresponding to the peak of the \( G_3 \) curve, produces the value of \( F_2 \) of only 0.19. This, of course, is a rather extreme example.

172. An unstated assumption in this Section’s discussion is that the private actor’s interpretation of an uncertain standard does not influence the government’s own interpretation of it. One can imagine situations where this assumption would not hold, that is, where market practice affects or even constrains the government’s interpretation.

173. See Sam Young & Lee Sheppard, Korb Slams Textron Ruling, Wall Street Rule, 117 TAX NOTES 204 (2007). According to one IRS official, the “Wall Street Rule” states that “the IRS can’t attack the tax treatment of a transaction if there is a long-standing and generally accepted understanding of its expected tax treatment.” Heather Bennett, Parker Debunks “Wall Street Rule,” Pushes LTR Preconferences, 100 TAX NOTES 1634, 1634 (2003).

174. See SEC Staff Accounting Bulletin No. 99, 64 Fed. Reg. 45,150, 45,151 (Aug. 19, 1999) (reminding practitioners that the term “material” may not be interpreted to exclude a “misstatement or omission of an item that falls under a 5% threshold,” as “one rule of thumb in
recognize the “5-10 percent price increase for one year or longer”\textsuperscript{175} rule of thumb, they are quite clear that these percentages are not set in stone.\textsuperscript{176} In all these and similar cases, the government regulators insisted—wisely—that vague legal provisions do not become more certain or more lenient over time merely because practitioners repeatedly interpret them in a particular way. A regulator may also counter the negative compliance effects of the market for legal advice by converting an uncertain standard into a clear rule, as the IRS did in its third and most recent set of anti-inversion regulations.\textsuperscript{177}

\textbf{F. The Likely Range of Compliance Efforts}

Some of the conclusions discussed above rely on the assumption that rational agents whose compliance efforts are observed by a regulator are likely to choose compliance efforts greater than $x^\theta$. Why would this be the case?

The most general assumption about the shape of legal uncertainty made here is that the density function has a single peak. Given this assumption, it is theoretically possible that rational actors take uncertain positions either exceeding or falling short of $x^\theta$. The latter outcome, however, appears to have a limited significance. In fact, it is difficult to identify a set of parameters that would produce this result under this Article’s model with a normally distributed legal uncertainty bound by a [0,1] interval.

This outcome is consistent with both C&C’s and Shavell’s findings. Shavell concludes that legal uncertainty induces actors to take more than due care as long as the distribution is not “too dispersed.”\textsuperscript{178} If the distribution representing legal uncertainty is single-peaked and symmetric, this means that compliance efforts will exceed $x^\theta$ if uncertainty is not too high. C&C’s simulations illustrate what “too dispersed” may mean. Many of their results yield over-deterrence (equivalent to $x^*$ above $x^\theta$ in the present specification), confirming this Section’s conclusion. Some C&C’s simulations reveal under-deterrence. However, almost all of the under-deterrence results arise from unrealistically high levels of uncertainty.\textsuperscript{179} If we focus only on C&C’s simulations where this problem does not arise, the vast majority of their results

\textsuperscript{175} Wegener, supra note 35, at 146.
\textsuperscript{176} See U.S. DEP’T OF JUSTICE & FED. TRADE COMM., HORIZONTAL MERGER GUIDELINES 8-10 (2010) (emphasizing that the FTC and DOJ may “use a price increase that is larger or smaller than five percent” as “small but significant” in defining the relevant product market).
\textsuperscript{177} See Treas. Reg. § 1.7874-3 (adopting a highly specific test based on several clear numerical thresholds to enable taxpayers to interpret the term “substantial business activities” for acquisitions completed on or after June 3, 2015).
\textsuperscript{178} SHAVELL, supra note 1, at 97.
\textsuperscript{179} I mean “unrealistically high” in the sense discussed in the text accompanying notes 130-131. To illustrate, consider the point in Table 1 where greater uncertainty begins to produce under-deterrence (in this model’s terms, $x^*$ drops below $x^\theta$). This point corresponds to the standard deviation of 400. Notably, the probability of success curve reflecting this standard deviation starts at 10.6%. A normal distribution is not a plausible representation of legal uncertainty in this case.
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correspond to agents taking better than fifty-fifty positions in terms of this Article’s framework.

Finally, the conclusion that private parties tend to take better than more-likely-than-not positions is supported by real-life observations. For instance, corporate reorganizations involving public companies often involve legal uncertainty regarding their tax-free status. These reorganizations also face inevitable scrutiny by the IRS. Companies routinely request their law firms to deliver legal opinions supporting the tax-free nature of the deals. The degree of confidence that shareholders demand in this context is typically either a “will” or a “should” level. The former amounts to something like an 85%-95% probability of success (0.85 < \( F(x) \) < 0.95); the latter to a 65%-85% probability (0.65 < \( F(x) \) < 0.85). At the same time, public companies rarely, if ever, proceed with transactions when their lawyers can only deliver a “more likely than not” (\( F(x) \) just in excess of 0.5) or a “substantial authority” (0.3 < \( F(x) \) < 0.5) opinion. These observations are consistent with the range of compliance efforts suggested by this Article’s model. They are also consistent with C&C’s analysis to a greater extent than it would appear from observing C&C’s reported simulation results. In a vast majority of simulations, the privately optimal compliance effort \( x^* \) indeed exceeds \( x^\prime \), and the resulting probability of success exceeds 50%.

G. The Effects of Uncertain Detection

Until this point, the discussion assumed that every uncertain position or transaction is scrutinized by the relevant authority. Needless to say, this is not always the case. For every high-profile tax-free deal guaranteed to attract attention from the IRS, there are numerous transactions that are unlikely to be audited or identified on audit. For every merger subject to a clearance by the FTC and the DOJ, there are many more potentially anti-competitive acts that are all but certain to escape the regulator’s gaze. All of this is well known in law and economics. In fact, detection uncertainty has been the focus of the economic analysis of law enforcement since its inception.

Modeling detection uncertainty in the presence of legal uncertainty is straightforward, at least in the first approximation. Detection uncertainty

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180. See, e.g., Williams Corp., Inc. v. Energy Transfer Equity, L.P., 2016 WL 3576682 (Del. Ch. June 24, 2016) (proposed merger requiring a “should” opinion about the merger’s tax consequences as a condition to closing).

181. See id.; see also Aabaco Holdings, 2015 Preliminary Information Statement (Exhibit 99.10) 45 (Nov. 16, 2015), http://www.sec.gov/Archives/edgar/data/1646775/000119312515378561/d77711dex991.htm (describing a “will” opinion from the company’s tax counsel in support of a tax-free spinoff).

182. See Raby & Raby, supra note 108.

183. See Polinsky & Shavell, supra note 40, at 412-13 (summarizing the literature).

184. I do not address here the possibility that detection and legal uncertainty may be interrelated. Such interrelationship is possible, see Richard Craswell, Deterrence and Damages: The
means that an actor may capture the benefit b no matter how implausible his legal position is if the regulator does not observe the position. The most important implication of introducing detection uncertainty is clear from Figure 5. When the likelihood of detection p is sufficiently low, an incentive to incur any cost of taking an uncertain legal position disappears. The specifics depend on the model parameters, but there is little doubt that if the likelihood of detection falls below, say, 10% (curve $G_3$), the model predicts that rational actors will simply evade the law.

Figure 5: The Effect of a Change in the Probability of Detection on the Gain Function

This result is well-known in the models that posit a world without legal uncertainty. The point here is that introducing legal uncertainty does not change the effects of uncertain detection.

The standard response to the imperfect detection problem is also well-known—the so-called penalty multiplier. If the penalty is based on the benefit

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185. More precisely, $p$ is the probability that a legally uncertain position will be scrutinized (for example, both audited and detected on audit). I treat this probability as exogenous, but this is not always the case. See Yitzhaki, *supra* note 68.

186. The algebraic form of the gain functions depicted in Figure 5 is $G(x) = (1 - p)(b - C(x)) + p(bF(x) - C(x)) = (1 - p)b + pbF(x) - C(x)$. In other words, by taking a position $x_i$, the actor incurs the cost of $C(x_i)$ (the last term), captures the gain of $b$ if the position is not scrutinized (the first term), and obtains the gain of $bF(x)$ otherwise (the second term).
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derived by the actor and includes this benefit, the multiplier equals \( \frac{1-p}{p} \).\(^{187}\)

Adding a penalty based on this multiplier to the gain function depicted in Figure 5 eliminates the effect of uncertain detection.\(^{188}\) In other words, the standard multiplier “works” in the presence of legal uncertainty as it does in its absence. So if a policymaker prefers more compliance to less, a detection-based multiplier is a good idea.

This conclusion begs the question. Should the two types of uncertainty be treated the same? Should the multiplier offset only detection uncertainty, or legal uncertainty as well? C&C assume that a multiplier should be set to reverse the effects of all uncertainty, including the legal one,\(^{189}\) and there appear to be no contrary arguments in the literature.

This conclusion makes sense in the optimal deterrence framework. Reversing the effects of uncertain liability for external harms makes them certain in expectation, forcing actors to take full account of the harms they produce. In the expected value terms, this converts a threshold-based regime (such as the “reasonable person” standard\(^{190}\)) into a strict liability regime. As is well-known, both strict liability and threshold-based regimes such as negligence may be socially optimal.\(^{191}\)

Although social optimality is irrelevant in the probabilistic compliance framework, incentives still matter. Reversing the effect of legal uncertainty means imposing a penalty that would offset the entire expected benefit from taking an uncertain position, no matter how strong that position happens to be.\(^{192}\) Just as the detection-related multiplier makes a private actor feel like a


188. To reflect the multiplier-based penalty, we would add a new term to the equation in footnote 186. It would reflect that if the uncertain position is detected (with probability \( p \)) and legal uncertainty is resolved against the actor (with probability \( (1-F(x)) \)), the actor will pay a penalty equal to \( \frac{1-p}{p} b \). If we reorganize and simplify the resulting equation, we will end up with the original equation for the gain function specified in supra note 48 as follows: \( G(x) = (1-p)(b-C(x)) + p \left[ b \cdot F(x) - C(x) \right] \).

189. See Craswell & Calfee, supra note 1, at 292-93 (assuming that the standard multiplier is the inverse of the probability of punishment, which reflects both detection and legal uncertainty). Gary Becker’s foundational work appears to take the same view. See Gary S. Becker, Crime and Punishment: An Economic Approach, 76 J. POL. ECON. 169, 174 (1968) (defining \( p \) as the probability of conviction, not just detection).

190. See Craswell & Calfee, supra note 1, at 292 n.18, 296 n.23.


192. Recall that when uncertainty relates to detection, the multiplier is \( \frac{1-p}{p} \). Because \( p \) is the probability that a penalty will be imposed (bad outcome) while \( F(x) \) is a probability that a penalty will not be imposed (good outcome), we need to substitute \( (1-F(x)) \) for \( p \) in the multiplier formula when we turn from detection to legal uncertainty. Thus, the multiplier needed to offset legal uncertainty is \( \frac{1+(1-F(x))}{1-F(x)} = \frac{p}{1-p} \). Ignoring detection uncertainty to simplify the exposition, and adding a
bad result (detection) is assured; the legal uncertainty-based multiplier makes a private actor feel like a bad result (finding of violation) is assured. Thus, an uncertain legal standard combined with a multiplier-based penalty aimed at offsetting legal uncertainty would amount to a complete prohibition. No rational lawmaker interested in compliance would approve such a regime. After all, if the policymaker wanted to enact an outright prohibition, it would have done so without bothering with devising a vague standard.

Thus, legal and detection uncertainty have different normative significance in this Article’s compliance-focused framework. Legal uncertainty is a design feature; detection uncertainty is a problem to be rectified. Whatever one thinks about using multipliers in order to offset imperfect detection, those reasons are inapplicable when we think about responding to uncertain, non-optimal laws.

It is worth noting that, when the proposed model incorporates detection uncertainty, it becomes inconsistent with real-life observations. In tax, for instance, the probability of detection (whether actual or perceived) is significantly below 10% for many transactions and positions subject to uncertain legal standards. The theoretical multiplier-based penalties for these kinds of detection probabilities are very high. The actual civil tax penalties are dramatically lower than the multiplier-based ones. Moreover, one can take a very weak yet plausible position (very low but positive value of x) and escape the specter of criminal sanctions altogether. If we assume that jail time, no matter how unlikely, is an enormous penalty that taxpayers want to avoid at all costs, the model predicts that they would take positions that will assure their freedom from criminal sanctions and nothing more. Yet numerous taxpayers, including highly sophisticated ones, choose to take much less aggressive positions regarding items that are very unlikely to be scrutinized by the IRS.

The model offered here cannot explain this behavior. In fairness, the standard economic analysis of compliance in the absence of legal uncertainty is not only equally incapable of providing the explanation, but it does not even recognize the problem of failing to deal with ambiguous legal standards.

Thus, even this model’s shortfall is the flip side of its conceptual advances.

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G(x) = bF(x) - C(x) - \frac{rG}{1 - (1 - F(x))} b(1 - F(x)) = -C(x). 
\]

193. See Alex Raskolnikov, Revealing Choices: Using Taxpayer Choice To Target Tax Enforcement, 109 COLUM. L. REV. 689, 701-02 (2009) (summarizing the literature on the actual and perceived probability of detection by the IRS).

194. See id.

195. See BITTKER & LOKKEN, supra note 58, ¶ 114.9.1 n.28.

196. As many tax practitioners would attest, the taxpayer’s goal is often to avoid all penalties, not just criminal ones.

197. The familiar puzzle from that analysis is why taxpayers comply rather than evade. This puzzle reflects the binary decision facing taxpayers in standard enforcement models that do not recognize legal uncertainty.
III. Expanding the Model

Every model has limitations, and the one presented here is no exception. This model relies on four main assumptions: a fixed benefit, a single-peak distribution as a reflection of an uncertain legal command, a smoothly increasing cost of compliance, and risk-neutral actors. These assumptions appear reasonable in many settings. Yet each may be questioned. Relaxing any of the assumptions makes the model more realistic and broadly applicable. But as this Part demonstrates, relaxing some of the assumptions dramatically complicates the model, making it difficult to identify even the basic relationships.

A. Complicating the Benefits

The vignettes used to motivate the basic model introduced in Part I are realistic, yet they do not capture many real-life scenarios. To start, each vignette reflects the assumption that the actor’s benefit is fixed. This need not be the case. The benefit may vary in many ways. Most importantly for the purposes of the model, the benefit may vary with \( x \).

Consider again the corporate law vignette involving the board of directors deciding how many costly fairness opinions to obtain in order to defend a transaction in court. The vignette posited that getting fairness opinions is costly, but this cost has nothing to do with the benefit from the potential corporate transaction. There are enough friendly bankers, the vignette suggested, to render as many board-friendly opinions as the board is willing to pay for. But it may well be the case that the more opinions the board solicits, the greater the chance that an opinion will conclude that the transaction is not fair at the price preferred by the board. In these and many other cases, a greater compliance effort \( x \) yields both a greater cost and a smaller benefit. In terms of the model, the benefit is no longer a constant, \( b \). Rather, it is a function \( B(x) \).

Relaxing the fixed benefit assumption makes the model even more similar to C&C’s and Shavell’s specifications. As discussed throughout, C&C’s simulations produce results that are similar to those generated by the basic model offered here, especially if we disregard the results arising from the implausible levels of uncertainty. Thus, extending the model to include variable benefits is unlikely to undermine the Article’s key findings.\(^{198}\)

\(^{198}\) An interesting special case of a variable benefit function is a benefit that is proportional (directly or inversely) to \( x \). This specification describes all sorts of valuation problems—an important subset of choices under uncertainty in several areas of the law. A detailed analysis of uncertain valuations is beyond this Article’s scope.
B. Complicating the Costs

The basic model assumes not only that the cost of compliance increases with $x$, but also that it does so gradually. While this assumption is standard in economic models, many realistic cost functions are not gradual.

Consider, again, a company about to engage in a tax-free sale and facing the “substantially all” test. Recall that the lawyers advise the company that the most likely meaning of “substantially all” is somewhere around 60%. It is quite plausible that the company would want to sell half (or some other fraction) of its assets in any case—but not more than that. So when the company is considering how much costly compliance to undertake, the company’s cost is zero for $x$ between 0 and 0.5, and is positive after that. In other words, the cost curve may have a kink at 0.5, and it may even have a notch—a jump from zero to a value meaningfully different from zero—at this point.

Things may be even more complicated. Perhaps the company faces some non-tax regulatory hurdles if the percentage of assets transferred exceeds a particular threshold. This introduces another kink (or notch) in the cost function. Many similar scenarios may be easily envisioned for other vignettes introduced at the beginning of the Article. More generally, cost functions may not be smooth, and may have all sorts of discontinuities.

This would present problems for the analysis. If the cost function has kinks and notches, it is impossible to solve the model algebraically. Simulations become more difficult as well, and highly sensitive to the location and the magnitude of notches and kinks. All of this weakens the model’s predictive power. Recall, for instance, that the model predicts that rational actors would rarely take positions that have less than a fifty-fifty chance of being upheld on review. Kinks and notches may cause a privately optimal compliance effort to be smaller than $x^p$. In fact, to the extent that rational actors are observed taking aggressive uncertain positions that are more likely to be wrong than right, a plausible explanation consistent with the model is that these actors have non-gradual cost functions.

C. Complicating the Actor’s Position

The actor’s compliance effort in the basic model reflects a variation along a single dimension. The $x$-axis reflects a percentage, a time period, a quantity, or some other number. This assumption reflects plenty of real-life legal standards, but certainly not all. The law is full of multi-factor tests. In tax, for instance, a legal expert needs to balance as many as ten or more different

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199. More precisely, the model assumes that the cost of compliance is differentiable.
200. This, for example, will be the case whenever the compliance effort gives rise to a fixed cost. A notch is a jump in a function, while a kink is a sharp turn. See Joel Slemrod, Buenas Notches: Lines and Notches in Tax System Design, 11 J. TAX RES. 259, 259 (2013).
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factors to decide whether a security is debt or equity, whether a worker is an employee or a contractor, and so on. Can a single dimension (x) represent such multi-factor analysis?

It can. To see how, imagine a two-factor test. For instance, assume that whether a security is debt or equity for tax purposes depends on its maturity and subordination. Because interest on debt is deductible but dividends on stock are not, a corporate issuer would prefer a debt characterization. However, the issuer would also prefer a longer maturity and deeper subordination as a matter of economics, and both features make security more equity-like. Thus, shortening the term of the security and increasing its seniority is costly, but also makes the debt characterization more likely.

How would an issuer choose the best maturity-seniority combination? It would start by ascertaining the cost of each combination. It would then arrange these combinations in order of increasing costs (with more than one combination possibly producing the same cost). Finally, the issuer would evaluate the cost-benefit tradeoff along the lines suggested by this Article’s model. Importantly, the vector of increasingly costly maturity-seniority combinations is the present model’s x-axis.

Needless to say, this exercise becomes exceedingly complicated when the number of factors increases. But greater complexity only means that real-life decision-makers do not engage in this exercise with a great degree of precision. At the same time, there is no doubt that these decision-makers and their legal advisors do evaluate the very tradeoffs exemplified in the two-factor example every time they make decisions in the presence of an uncertain multi-factor test. Thus, accounting for more complex positions does not undermine the present model.

D. Incorporating Risk Preferences

Finally, the model assumes risk-neutral actors. This assumption is not always plausible. Introducing risk preferences raises several difficult questions.

First, moving along the x-axis changes both the expected value and the degree of risk incurred by the private actor. The standard way of investigating such compound changes is to disaggregate them into a riskless variation of the expected value and a series of zero-sum risky bets. Up to this point, the model focused only on the first element of this combination—the expected value variation—because risk-neutral actors have no preferences over different

201. See BITTKE & LOKKEN, supra note 58, ¶ 91.10.2.
202. See id. ¶ 111.5.2.
203. For a discussion of these factors, among many others, see I.R.S. Notice 94-47, 1994-1 C.B. 357.
204. See LOUIS ECKHOUDT et al., ECONOMIC AND FINANCIAL DECISIONS UNDER RISK 7 (2005).
zero-sum bets. Once risk-averse actors enter the picture, we must address the second element.

Doing so presents a challenge. The basic approach to modeling the cost of risk is the so-called mean-variance model. That model is based on an approximation that establishes a proportional relationship between the cost of risk measured by the risk premium and the dispersion of risky payoffs. That approximation, however, is accurate only for small risks or in special cases where the entire distribution may be fully described by its mean and variance. More generally, risk premium depends on the properties of a probability distribution other than its mean and variance, and, relatedly, on higher order derivatives of the actor’s utility function. Recent experimental research suggests that utility functions traditionally used to model risk aversion may fail to reflect people’s higher-order risk preferences. None of this work has been applied to the analysis of legal uncertainty as far as I know.

Second, the uncertainty just discussed relates solely to the chance of losing the benefit. However, many uncertain rules are also risk-based. For example, a rule may require the taxpayer to retain an uncertain amount of unwanted risk of loss. In contrast with the risk discussed in the previous paragraph, the risk resulting from risk-based rules increases monotonically (though not necessarily linearly) with $x$. The model’s cost function $C(x)$ already reflects this kind of cost. However, when $C(x)$ is understood as arising from the risk imposed by risk-based rules (rather than fees paid to bankers or expenses of selling assets) it may be possible to reflect some of the standard assumptions about risk tolerance—such as the constant relative risk aversion—in the expected utility calculations.

Finally, the probability of success that lawyers convey to a client for a particular value of $x$ is itself uncertain. This is the so-called ambiguity—lack of clarity about the probability that a particular event will occur. Importantly, the extent of ambiguity surrounding uncertain legal commands varies with $x$. For values of $x$ corresponding to very small and very large values of $F(x)$ the

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205. See id.
206. This is called the Arrow-Pratt approximation. See id. at 11.
207. Examples of such distributions include normal, lognormal, and gamma distributions.
208. More precisely, the premium depends on all four central moments of a distribution. See ECKHOUDT ET AL., supra note 204, at 12.
210. See id. at 1416.
212. For a discussion of constant relative risk aversion utility functions, see, for example, ECKHOUDT ET AL., supra note 204, at 67.
213. See Talley, supra note 85, at 763-65. When the probability judgment is subjectivist, ambiguity refers to the confidence in one’s subjective judgment about a future event.
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ambiguity is small; otherwise it may be considerable. For example, when a lawyer advises a client that there is a 95% chance of obtaining a benefit, the lawyer is likely to be fairly confident in the 95% level. If nothing else, this number has a clear and not-too-distant upper bound. In contrast, when a lawyer states that obtaining a benefit is a 50-50 bet, the lawyer may admit that his confidence in the 50-50 number is not that high, so the bet may be 40-60 or 60-40. Needless to say, a client may view such ambiguity as an additional cost.

In sum, introducing risk- and ambiguity-related preferences into the model complicates the analysis considerably, but promises novel and interesting insights. Research into these topics has barely begun.214

IV. Probabilistic Compliance and the Complete Deterrence Theory

Until now, we have contrasted the probabilistic compliance approach offered here to the theory of optimal deterrence. Although this theory is indeed the dominant economic approach for evaluating legal regimes, it is not the only one.

The alternative approach developed by legal economists is that of complete (or absolute) deterrence.215 While optimal deterrence focuses on maximizing efficiency, complete deterrence aims to ensure compliance. Optimal deterrence generally achieves its goals by forcing actors to internalize the external harms of their acts.216 Complete deterrence aims to deny violators all gains from their violations.217

The complete deterrence theory is not a legal economist’s first choice. Its normative appeal is weak—why insist on enforcing bad laws? Its gain-based sanctions may lead to large social losses if courts make even small mistakes in assessing the offender’s gain—a deficiency that the optimal deterrence approach does not share.218

The appeal of the complete deterrence theory is practical, not theoretical. In contrast with the optimal deterrence theory, the complete deterrence theory does not rely on an unrealistic assumption of efficiency-maximizing rules. Moreover, numerous statutory sanctions in criminal law, securities regulation, antitrust law, and tax law, to take some examples, explicitly aim at denying the

216. See Polinsky & Shavell, supra note 40, at 408.
217. See Hylton, supra note 215, at 421.
218. See A. Mitchell Polinsky & Steven Shavell, Should Liability Be Based on the Harm to the Victim or the Gain to the Injurer?, 10 J.L. ECON. & ORG. 427, 432 (1994).
offenders gains from their violations. These gain-based sanctions strongly link the complete deterrence theory to real-life regulatory regimes.

Despite this appealing connection, the complete deterrence theory has had a decidedly limited reach. Perhaps in response to the doubts about the theory’s normative foundations, its proponents usually limit their analyses to acts that may not plausibly be viewed as socially desirable. Violent crime, tax evasion, and egregious harms deserving punitive damages are the typical examples. Not surprisingly, the complete deterrence literature has ignored legal uncertainty. If the goal is to deter an activity completely, any uncertainty that potentially permits the activity is undesirable.

Yet plenty of legal uncertainty exists in regulatory regimes that rely on gain-based sanctions and do not address egregious, reprehensible, violent acts. For instance, while criminal tax evasion is subject to gain-based fines, so are the tax positions that are possibly legal but ultimately found to be incorrect. Gain-based sanctions also apply to civil securities law violations, civil environmental violations, and many other transgressions that by no means arise from activities that should be obviously deterred altogether. Do gain-based sanctions make sense for these less-than-absolutely undesirable activities? What incentives do these sanctions create when they enforce uncertain laws? Most generally, what can the economic analysis tell us about gain-based sanctions if we abandon the all-or-nothing complete deterrence objective and admit the reality of legal standards that are both uncertain and non-optimal?

As is probably obvious by now, these are the questions that this Article begins to answer. Recall that the bulk of the discussion assumes that the only consequence of the actor’s noncompliance is losing the benefit. This is precisely what a gain-based sanction is. Thus, this Article extends the economic

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219. For example, the penalty for insider trading is based on “the profit gained or loss avoided,” 15 U.S.C. § 78u-1(a)(2) (2012), the Criminal Fines Improvement Act penalizes many white-collar crimes by fines that are “not more than the greater of twice the gross gain or twice the gross loss” captured or caused by the offender, 18 U.S.C. § 3571(d), and the accuracy-related tax penalties are based on the amount of tax underpayment, I.R.C. § 6662(a).

220. See Hylton, supra note 215, at 422; David Markell, Is There a Possible Role for Regulatory Enforcement in the Effort To Value, Protect, and Restore Ecosystem Services?, 22 J. LAND USE & ENVTL. L. 549, 566 n.76 (2007).

221. This discussion does not refer to the literature exploring how gain-based sanctions may support optimal deterrence. That literature shares the objective of welfare maximization and sometimes expressly assumes that the law is optimal. For recent examples, see Robert Cooter & Ariel Porat, Disgorgement Damages for Accidents, 44 J. LEGAL STUD. 249, 256 (2015); Bert I. Huang, The Equipoise Effect, 116 COLUM. L. REV. 1595, 1600 (2016). In contrast, the present inquiry and the complete deterrence theory abandon both the objective and the assumption described above.

222. See I.R.C. § 6662(d)(2)(B) (imposing a penalty on undisclosed positions lacking “substantial authority” and disclosed positions lacking “reasonable basis”).


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analysis of gain-based sanctions to settings where law is uncertain and where complete deterrence is neither plausible nor desirable. Perhaps, we should start referring to gain-based (rather than complete) deterrence, and investigate its implications for a much broader range of violations than the complete deterrence theory has studied thus far. The probabilistic compliance perspective offered here would allow us to do just that.

Conclusion

This Article investigates a familiar model of legal uncertainty from a new perspective. It does so by shifting the focus from deterrence to compliance, from the actor’s choice of effort to the probability of success of the actor’s position. This shift yields new insights. It also necessitates an inquiry into the meaning of compliance when the law is uncertain.

The Article’s main takeaways may be summarized as follows. First, the famously ambiguous relationship between legal uncertainty and deterrence does not extend to the relationship between legal uncertainty and compliance. Rather, greater certainty mostly leads to greater compliance. Second, greater certainty likely leads to higher private gains. Third, the market for legal advice tends to increase the perceived certainty of the law and make uncertain standards appear more permissive. Both effects induce less compliant, more aggressive positions and increase the number of actors choosing to take them. Fourth, the standard penalty multiplier offsets detection uncertainty whether or not legal uncertainty is present. However, incorporating the multiplier into the model reveals why it should not be used to offset legal uncertainty.

The Article evaluates these findings from the perspective of a regulator who recognizes that compliance is probabilistic, prefers more compliance to less, but does not demand perfection. Even this loosely specified regulatory objective is inconsistent with some common government enforcement strategies in view of the model’s results. Regulators should rethink their reliance on the in terrorem effect of vague standards when the actions of regulated parties are observable. Regulators should also be wary of leaving uncertain laws on the books without either clarifying them or reiterating their uncertainty.

Finally, the Article offers a path to a significant extension of the complete deterrence framework that has long played a second fiddle to the dominant optimal deterrence theory. The framework’s distinctive feature is gain-based sanctions—the same sanctions that are built into the present model. However, in contrast with the complete deterrence framework, the probabilistic compliance approach extends the analysis of these sanctions to settings where the law is uncertain, and where complete deterrence is not appropriate or achievable. Granted, even with this extension, the probabilistic compliance framework does not offer a strong normative justification of gain-based sanctions. But understanding their incentive effects in the presence of legal
uncertainty should be quite useful to regulators grappling with the day-to-day realities of enforcing countless existing laws, uncertain and imperfect as they may be.