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A Note on Presumptions with Sequential Litigation*

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Abstract

This note extends the Bernardo, Talley & Welch (1999) model of presumptions to study situations where litigation efforts are spent sequentially rather than simultaneously. The equilibria of the litigation stage are presented as functions of b. Two characteristics distinguish this case from the simultaneous one. First, sequentiality allows the principal to pre-commit to a litigation strategy, and thus possibly preempt litigation effort by *both* agent-types. Second, while the description of the equilibria is more complex (and must be divided into four regions of b), the comparative statics that emerge are somewhat simpler than in the simultaneous case.

1 Introduction

This technical note solves the conflict model in Bernardo, Talley & Welch (1999), but with sequential rather than simultaneous litigation expenditures by the plaintiff and defendant, respectively. In particular, assume in what follows that if a principal sues, she must first "make out a case" by presenting her evidence L_P ahead of the agent. Only after observing L_P does the agent proceed to make his case in his own defense (*i.e.*, L_A^H or L_A^L, depending on his type).

2 A Conflict-Theory Model of Agency Costs

In this section, we develop a conflict-theory approach to characterize the role of legal presumptions in a standard agency model. Our model begins with a simple

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moral-hazard framework. However, once the agent has chosen an action, we introduce an explicit litigation stage. The primary parameter of interest occurs within this latter stage, in the form of a legal presumption specifying the manner in which courts weigh and process each party's proffered evidence so as to reach a decision. As we demonstrate in subsequent sections, the strength of the presumption provides an important link between productive and redistributional incentives.

2.1 Framework

Consider a two-person game involving a principal ("she") who hires an agent ("he") to provide labor for some productive enterprise (the "project"). In performing his duties, the agent is assumed to make a private, non-monitorable decision about whether to expend high effort (e^H) or low effort (e^L). Although it costs the agent nothing to expend low effort, high effort imposes on him a non-monetary cost of ϕ dollars. Nevertheless, a high level of effort can benefit the principal, as it affects the probability that the project realizes a high payoff (V_H) instead of a low payoff (V_L, where $V_L < V_H$). In particular, the relationship between the agent's effort choice and the likelihood of project success is summarized in the following table:

	High (V_H)	Low (V_L)
Effort	p	$1-p$
No Effort	$1-p$	p

(1)

The parameter $p \in [^1/_2, 1]$ captures the degree to which the agent's effort can affect prospective outcomes, with larger values representing a greater importance of effort on the project's success rate. In general, because $p \geq 1/2$, the principal—who is the residual claimant on the project's revenues—would always like the agent to choose a high effort level. (From a societal standpoint, of course, effort is desirable only if $(2p-1)(V_H - V_L) > \phi$). The principal observes only the project's outcome—she is unable to observe the agent's actual effort choice directly.[1]

In most standard agency-cost models, the optimal contractual solution is to offer incentive pay: i.e., the principal promises to pay the agent a bonus should the project yield a high payoff. In contrast to this standard approach, we limit our attention below to "fixed-wage" contracts, in which the agent receives a wage of w regardless of the realized state, but may be subject to suit should a low project payoff obtain. We motivate this assumption on a number of grounds. First, our

[1] We also assume that the agent's effort is not verifiable (at least directly) in court. Nonetheless, as we demonstrate below, the underlying evidentiary rule may act as an *indirect* means of verifying the agent's effort (at least probabilistically).

principal focus concentrates on the use of default legal rules (rather than express contract terms) as a means for providing optimal incentives for the agent. Although such rules utilize "sticks" rather than "carrots" as the primary tool for achieving incentive compatibility, an efficient default rule can substitute for express terms, which themselves are often costly or difficult to draft.[2] On a related note, so long as express incentive contracts have at least some enforcement costs (*e.g.*, they require that a court stand ready to verify whether a low or high state has obtained), one can specify a default rule within our framework below that is an *exact* substitute for an incentive contract. Finally, there are (for reasons outside our model) a number of contract doctrines that are immutable in nature, and thus preclude private parties from contracting around them.[3] In such situations, litigation may be the sole mechanism for enforcing contractual allocations.

Returning to our model, then, we assume that the agent receives a constant wage w, but if a low state obtains the principal may file a lawsuit against the agent. The lawsuit—if successful—would require the agent to pay money damages to the principal. Accordingly, the extensive form of the game is as follows:

[INSERT EXTENSIVE FORM HERE]

The figure presupposes that the agent has been offered and accepted a contract paying him a specified wage, w, which satisfies his participation constraint. The agent is first to move, deciding whether to expend high or low effort in performing his duties. Next, Nature determines whether the project yields a high or a low payoff, according to the probabilities associated with the agent's effort choice. Should the project yield a high payoff, the game immediately ends, with the high-effort agent type receiving a payoff of $w-\phi$, the low-effort agent type receiving a payoff of w,and the principal receiving a payoff of $V_H - w$.

Should a low payoff obtain, the principal may choose whether to file suit against the agent. If she decides not to sue (NS), then the game similarly ends

[2]There is a growing literature on the costs of express contracting, costs that emanate from problems of (among other things) bounded rationality, multi-tasking concerns, complexity, and intra-organizational political concerns. See, e.g., Holmstrom & Milgrom (1991, 1994); MacLeod (1998); Jensen & Murphy (1990).

[3]A number of statutes make certain types of contracts invalid. See, e.g., Cal. Civ. Code § 1668 (1999) (voiding as unlawful all contracts that exempt anyone from responsibility for fraud, willful injury of another, or violation of law). In some jurisdictions, this trend appears to be continuing. See, e.g., California Assembly Bill 858, 1999 CA A.B. 858 (voiding, as against public policy, all contracts in which consumers or employees consent to binding arbitration, waive their right to rescind a contract during a statutory cooling-off period, or waive their rights to a jury trial). In this paper, however, we do not attempt to provide a reason for immutable rules, other than to recognize that they exist in many circumstances.

with the high-effort agent, low-effort agent and the principal, respectively, receiving payoffs of $w - \phi$, w, and $V_L - w$. If, on the other hand, the principal decides to sue (S), the players enter an end game of litigation, in which a court must decide whether to find the agent liable. Should liability be found, the agent must pay D dollars to the principal, an amount representing the applicable damages for the complaint in question.[4] While such prospective recovery is attractive to the principal, litigation does not come without costs. Indeed, in order simply to bring suit, the principal must incur a non-recoverable fixed cost $F > 0$ to draft and file a complaint. Thus, only if the expected net payoffs from litigation are sufficiently large to cover these fixed costs would a rational principal ever choose to file suit.[5]

Once invoked, litigation may impose additional *variable* costs on both parties as they argue the case in court. In particular, we conceive of litigation as a redistributional conflict game, wherein parties expend "litigation effort" producing and presenting evidence before a judge or jury. Let $L_P \geq 0$ denote the amount of incriminating evidence the principal chooses to present against the agent in litigation. Similarly, let $L_A^L \geq 0$ and $L_A^H \geq 0$ denote the amount of exculpatory evidence the low-effort and high-effort agent types, respectively, choose to offer in their own defense. Following the usual rules of civil procedure, assume that the players' decisions in the litigation game are sequential, with the principal first presenting her case-in-chief followed by the agent who presents his defense.[6]

The litigation strategies, L_P, L_A^L , and L_A^H, are intended to summarize the efforts that litigants routinely expend to gather and present evidence to a court (such as eye-witness testimony, expert opinions, documentary evidence, laboratory tests, and the like). Importantly, we make no specific assumption about the inherent truthfulness of either side's evidence. Indeed, it may be genuine and contrived; unrehearsed or completely orchestrated. All that we require is that the evidence be costly on the margin for both parties to produce. In particular, we assume that the principal faces a (constant) marginal litigation cost of $c_P > 0$ to present L_P, so that his total evidentiary cost is $c_P L_P$. The agent also bears a (constant) marginal cost of presenting evidence, but we allow the agent's cost to depend on his type (*i.e.*, prior effort level). If the agent expended high effort, his marginal litigation cost is $c_A^H > 0$, and thus his total evidentiary cost is $c_A^H L_A^H$. If he expended low

[4]In order to concentrate on the role played by legal presumptions, we treat D as exogenous in what follows, assuming only that it is "large" enough to have a potential deterrent effect on the agent. *See* Section 2.5, *infra*. In principle, it is possible to generalize the model to allow for suit in either realized state of the world. Doing so, however, adds considerable complication to the analysis without many added insights. We have therefore omitted such an analysis in what follows.

[5]It is easy to demonstrate that the principal will *always* file a complaint if $F = 0$. Thus, we limit our attention to the (more realistic) case of $F > 0$.

[6]It is also possible to model the litigation game as simultaneous, and similar results emerge. *See* Bernardo, Talley & Welch (1999).

effort, his marginal litigation cost is $c_A^L > 0$, and thus his total evidentiary cost is $c_A^L L_A^L$. We assume in what follows that $c_A^L > c_A^H$; i.e., shirking agents find it more costly to produce exculpatory evidence than do their high-effort counterparts.[7] We justify this assumption by observing that shirkers must (almost by definition) offer evidence that is inconsistent with their actual behavior. As such, producing such evidence may necessitate exhaustive searches, more intensive coaching of friendly witnesses, and perhaps even the payment of explicit or implicit bribes in exchange for false testimony.[8] As will become apparent below, this cost differential implies that shirking agents will rationally choose to present less evidence than their non-shirking counterparts in equilibrium. Consequently, the litigation effort expended by the agent may be an efficiency-enhancing signal of her type—a signal that is only possible when litigation occurs along the equilibrium path.

Finally, in order to understand why the parties would even bother to expend litigation effort, it is important to specify how evidence presentation affects judicial findings of liability. To this end, let $q(L_P, L_A^j)$ denote the "legal rule" employed by the court, which maps the players' litigation efforts into the probability that the agent is found liable, with $j \in \{L, H\}$. (Alternatively, it is possible to interpret $q(.)$ as the fraction of some maximal damages amount D that the principal receives). In order to develop more concrete intuitions (and to remain consistent with the conflict-theory literature[9]), we adopt a particular functional form for $q(.)$, in which the principal's success probability is:

$$q_j \equiv q(L_P, L_A^j) = \frac{L_P}{bL_A^j + L_P} \Longleftrightarrow \left(\frac{q(L_P, L_A^j)}{1 - q(L_P, L_A^j)} \right) = \frac{L_P}{b \cdot L_A^j} \tag{2}$$

for $j = L, H$. (To economize on notation, in what follows we will often denote $q(L_P, L_A^j)$ simply as q_j). The parameter $b > 0$ denotes the *ex ante* "weight" that a court accords the agent's proffered evidence relative to the principal's, thereby representing the role of a legal presumption. Moreover, by varying the value of b it is possible to consider a range of potential presumptions, from a conclusive (or "irrebuttable") presumption favoring the principal ($b = 0$) to a conclusive presumption favoring the defendant ($b = \infty$), and all (theoretically rebuttable) presumptions in between ($0 < b < \infty$).[10]

[7] Although we assume constant marginal costs, most of the core arguments presented below carry over to the case in which the marginal cost of evidence production increases in litigation effort (so long as the low-effort agent's cost schedule is uniformly higher than that of the high-effort agent).

[8] Similar assumptions appear in Rubinfeld & Sappington (1987), Sanchirico (1998) and Daughety & Reinganum (1998). Daughety and Reinganum motivate their assumption a phenomenon of costly evidentiary search by "guilty" defendants.

[9] *E.g.,* Hirshleifer (1995).

[10] There are other possible evidentiary interpretations of the b parameter. For example, a judicial

This functional form exhibits a number of useful and intuitive properties.[11] First, note that it is increasing in L_P and decreasing in L_A^j: greater litigation effort by either party *ceteris paribus* increases her likelihood of prevailing (or alternatively, her share of the surplus available for redistribution). Moreover, it is possible for either party—holding her opponent's action constant—to choose a level of litigation that realizes the entire range of success probabilities between 0 and 1. Finally, as the two parties' litigation levels tend uniformly to zero, the limiting probability of plaintiff success is $1/(1+b)$,which one might interpret as the court's default presumption—*i.e.*, its *ex ante* bias in the absence of any production of evidence.[12]

3 Equilibrium Behavior

Given the fundamentals of the game, we may now proceed to analyze the plausible equilibria that emerge from non-cooperative play. Our equilibrium concept in what follows is *sequential equilibrium* (Kreps & Wilson 1982), though even weaker equilibrium concepts would do as well.[13] Denote the probability that the agent expends high effort by β, and the probability that the principal brings an action in a low state by γ. Accordingly, the strategy profile $(\beta^*,\ \gamma^*,\ L_P^*,\ L_A^{L^*},\ L_A^{H\ *})$ is part of a sequential equilibrium for the game if no player-type has an affirmative incentive to deviate from her prescribed strategy given her beliefs at each stage, and if all players' beliefs at each information set are consistent and sequentially rational. We solve the game in reverse order, starting with the litigation contest, then inducting backwards to the principal's decision about whether to file suit, and

bias toward a litigant may manifest itself in the relative frequency with which the court deems one party's evidentiary offerings inadmissibile. A pro-plaintiff court may tend to admit virtually all of a plaintiff's offers of proof, while admitting the defendant's only 70 percent of the time. Such a bias would be reflected by setting $b = 0.7$.

[11] In addition to those listed in the text, Skaperdas (1996) shows that this functional form also has some desirable axiomatic properties in other contexts, such as a monotonic improvement in outcome when more resources are expended. The only other known conflict paramaterization that satisfies such properties (exponential) leads to corner solutions.

[12] Explicitly, $\lim_{L\to 0} q(L,L) = 1/(1+b)$. As a formal matter, of course, legal rules on burdens of production may look slightly different. The conventional description (*e.g.,* Fuller (1967)) appears to be one of an absolute (rather than probabilistic) presumption, which oscillates between the plaintiff and defendant *ad seriatim* as each meets her burden of evidence production to overcome it. It is possible to capture such intuition by either assuming a more elaborate sequence of evidence production, or (in what may be a limiting case) assuming simultaneous actions. Such permutations appear not to alter our qualitative results. *See* Bernardo, Talley & Welch (1999).

[13] In particular, the set of equilibria described below is also the set of perfect bayesian equilibria (PBE), an equilibrium concept that does not require consistency in beliefs. The distinction between PBE and sequential equilibrium is non-existent for this model, as all relevant information sets are reached with positive probability.

finally to the agent's *ex ante* decision about whether to expend effort.

3.1 Litigation Stage

To analyze the final, litigation stage of the game, assume that a low state of the world has come about and that the principal has chosen to file suit. Let α denote the principal's belief that the agent has previously expended a high level of effort if the low state occurs.[14] The endogenous levels of litigation activity L_A^L, L_A^H, and L_P will generally depend on α, and are characterized below.

First consider the agent's choice of litigation level, having observed the principal's choice L_P. Because the agent knows how much effort he has previously put forth, his choice will generally depend on his type. For the agent type who previously put forth high effort (e^H), the problem is to solve:

$$\max_{L_A^H \geq 0} \left[\frac{L_P}{bL_A^H + L_P} \right] \cdot (-D) - c_A^H L_A^H. \tag{3}$$

Assuming an interior solution,[15] the following first-order condition characterizes the high-effort agent's best response to L_P[16]:

$$\left(\frac{bL_P}{(bL_A^H + L_P)^2} \right) \cdot D = c_A^H. \tag{4}$$

For an agent who put forth low effort (e^L), the analogous problem is:

$$\max_{L_A^L \geq 0} \left[\frac{L_P}{bL_A^L + L_P} \right] \cdot (-D) - c_A^L L_A^L. \tag{5}$$

Assuming an interior solution, the relevant first-order condition is:

$$\left(\frac{bL_P}{(bL_A^L + L_P)^2} \right) \cdot D = c_A^L. \tag{6}$$

The conditions (4) and (6) embody the intuition that a each agent type will expend litigation costs up to the point where the marginal benefit of reducing his expected

[14]Though α should be treated as exogenous at this stage, sequential rationality requires its value to be related to the agent's equilibrium effort choice β via Bayes rule. This constraint is taken up at length *infra* in subsection 2.5.

[15]It turns out that an interior solution for the high-effort agent need not exist. In particular, his optimal choice is interior if and only if $b > \frac{\alpha c_A^H}{2c_P}$.

[16]In this case, as in the others, sufficiency is satisfied by the strict concavity of the objective function in L_P.

liability is equal to the marginal cost of litigation effort. Note that these conditions (if satisfied at equality) imply that $q_i = \sqrt{\frac{c_A^i L_P}{bD}}$, for $i = L, H$.

We now move backwards in the sequence to consider the principal's choice of litigation effort. Having already sunk the fixed cost of bringing suit, the principal's expected payoff from litigation consists of damages she can expect (i.e., $[\alpha q_H + (1-\alpha)q_L] \cdot D$) less her variable costs of litigation ($c_P L_P$). Thus, given the respective agent types' litigation levels, the principal's optimization problem solves the following:

$$\max_{L_P \geq 0} \left[\alpha \left(\frac{L_P}{bL_A^H + L_P} \right) + (1-\alpha) \left(\frac{L_P}{bL_A^L + L_P} \right) \right] \cdot D - c_P L_P. \tag{7}$$

Substituting the agent-types' optimality conditions from (4) and (6) (again assuming that interior solutions obtain) and differentiating gives the following first-order condition for the principal:

$$\frac{1}{2} \sqrt{\frac{D}{L_P \cdot b}} \left(\alpha \sqrt{c_A^H} + (1-\alpha) \sqrt{c_A^L} \right) = c_P. \tag{8}$$

As with the agent, this condition states that the principal will increase her litigation efforts (L_P) until the marginal private benefits from doing so are just equal to the marginal private costs.

If both agents' optimal choices are interior, the unique equilibrium of the continuation game can be found by solving (8), (4), and (6) simultaneously. Unfortunately, such a solution exists only if the underlying presumption b is sufficiently large to induce both agent types to put up a defense in equilibrium.[17] When b is relatively small, (indicating a relatively strong pro-plaintiff presumption), one or both of the agent types may find it optimal simply to fold and not present any evidence (*i.e.*, $L_A^{i*} = 0$ for $i = L, H$). Accounting for these considerations yields the following lemma (whose proof can be found in the appendix):

Lemma 1 *The equilibrium litigation efforts of the principal and the two agent*

[17] In particular, it is easily confirmed that a fully interior solution requires $b \geq \left(\frac{\alpha\sqrt{c_A^H} + (1-\alpha)\sqrt{c_A^L}}{2c_P} \right) \sqrt{c_A^L}$.

types in b−space (along with the equilibrium probabilities of liability) are:

Interval	L_P^*	L_A^{H*}	L_A^{L*}	q_H^*	q_L^*
$b \in B_4$	$\frac{D \cdot \overline{\mu}^2}{4b}$	$\frac{D \cdot \overline{\mu}}{2b} \cdot \left[\frac{1}{\sqrt{c_A^H}} - \frac{\overline{\mu}}{2b} \right]$	$\frac{D \cdot \overline{\mu}}{2b} \cdot \left[\frac{1}{\sqrt{c_A^L}} - \frac{\overline{\mu}}{2b} \right]$	$\frac{\overline{\mu}\sqrt{c_A^H}}{2b}$	$\frac{\overline{\mu}\sqrt{c_A^L}}{2b}$
$b \in B_3$	$\frac{b \cdot D}{c_A^L}$	$\frac{D}{\sqrt{c_A^L}} \cdot \left[\frac{1}{\sqrt{c_A^H}} - \frac{1}{\sqrt{c_A^L}} \right]$	0	$\sqrt{\frac{c_A^H}{c_A^L}}$	1
$b \in B_2$	$\frac{D \cdot \underline{\mu}^2}{4b}$	$\frac{D \cdot \underline{\mu}}{2b} \cdot \left[\frac{1}{\sqrt{c_A^H}} - \frac{\underline{\mu}}{2b} \right]$	0	$\frac{\underline{\mu}\sqrt{c_A^H}}{2b}$	1
$b \in B_1$	$\frac{b \cdot D}{c_A^H}$	0	0	1	1

where $\underline{\mu} \equiv \left(\frac{\alpha\sqrt{c_A^H}}{c_P} \right)$ *and* $\overline{\mu} \equiv \left(\frac{\alpha\sqrt{c_A^H}+(1-\alpha)\sqrt{c_A^L}}{c_P} \right)$, *and* $B_1 \equiv \left[0, \frac{1}{2}\underline{\mu}\sqrt{c_A^H}\right)$, $B_2 \equiv \left[\frac{1}{2}\underline{\mu}\sqrt{c_A^H}, \frac{1}{2}\underline{\mu}\sqrt{c_A^L}\right)$, $B_3 \equiv \left[\frac{1}{2}\underline{\mu}\sqrt{c_A^L}, \frac{1}{2}\overline{\mu}\sqrt{c_A^L}\right)$, *and* $B_4 \equiv \left[\frac{1}{2}\overline{\mu}\sqrt{c_A^L}, \infty\right)$.

Notice that the only fully-interior equilibrium of the litigation game occurs when $b \in B_4$, an interval in which the underlying presumptions accords the defendant (of either type) sufficient leverage to make it worthwhile to attempt a defense. In any other interval, either the low-effort defendant or both types of defendant are at corner solutions, and therefore present no exculpatory evidence. Moreover, since a defendant who fails to mount any defense loses with probability one, there are no "false negatives" (i.e., low-effort defendants who are nonetheless exonerated) in $\{B_1, B_2, B_3\}$—only "false positives" (high-effort defendants who are nonetheless found liable). Interval B_4 is the only one in which there is a meaningful tradeoff between overinclusive and underinclusive legal rules (and thus B_4 will receive most of our attention below).

Inspection and/or piecewise differentiation of the above expressions yields the following results for the principal:

Lemma 2 *The principal's equilibrium litigation level,* L_P^*, *is increasing in* D, *and is globally maximal at either* $b = \frac{1}{2}\underline{\mu}\sqrt{c_A^H}$ *or at* $b = \frac{1}{2}\overline{\mu}\sqrt{c_A^L}$.

Note from the lemma that the principal is more likely to sue if, *ceteris paribus*, there is more at stake (i.e., D is large). One would expect this tendency, since the marginal value of litigation effort increases in D and the marginal costs are invariant to D. Furthermore, note that there is an interior b that induces the principal to be most litigious. This also comports with intuition: for when the principal has a conclusive edge (as is the case when b is extremely low), she need not expend much effort to challenge the plaintiff; conversely, when the agent has a conclusive edge (as when b is extremely high), it is futile for the principal to expend much on litigation, since she is likely to lose regardless. Only at an intermediate value of b are the parties chances of prevailing roughly commensurate. It is here, then, where the principal (and the agent) tend to be most aggressive in litigation (see Welch (1998)).

Analogously, a similar set of results can be obtained for both the high-effort and the low-effort agent types:

Lemma 3 *The high-effort agent's equilibrium litigation level, L_A^{H*}, is equal to zero for all $b \in \{B_1\}$. For all $b \in \{B_2, B_3, B_4\}$, L_A^{H*} is positive, strictly increasing in D, strictly greater than the low-effort agent's equilibrium litigation level, L_A^{L*}. Finally,*

- *If $c_A^H > \frac{c_A^L}{4}$, then L_A^{H*} is maximal at $b = \overline{\mu}\sqrt{c_A^H}$;*
- *If $c_A^H < \frac{c_A^L}{4}$, then L_A^{H*} is maximal at $b = \underline{\mu}\sqrt{c_A^H}$;*
- *If $c_A^H = \frac{c_A^L}{4}$, then L_A^{H*} is maximal $\forall\, b \in \{B_3\}$.*

Lemma 4 *The low-effort agent's equilibrium litigation level, L_A^{L*},is equal to zero for all $b \in \{B_1, B_2, B_3\}$. For all $b \in \{B_4\}$, L_A^{L*} is strictly positive and increasing in D. Finally, L_A^{L*} is maximal at $b = \overline{\mu}\sqrt{c_A^L}$.*

Like principals, agents are more inclined to defend themselves if more is at stake (D is large) and if their net "power" (taking into account judicial presumptions and costs of litigation) is roughly commensurate with that of the principal. Note that both the principal's and the agent's actions depend on the principal's equilibrium assessment that the agent has exerted effort (α). Although we treat α as exogenous for current purposes, subsequent sections will make clear that the principal's beliefs may be sensitive along the equilibrium path to variations in the value of b (i.e., a larger b tends to reduce agent effort, which in turn reduces the principal's Bayesian assessment of the agent's culpability).

It is also important to note that in all but interval B_1 (where neither agent type mounts a defense), the high-effort agent is a more aggressive litigator than is his shirking counterpart. This difference in litigiousness is an artifact of the marginal cost differences faced by the two agent types (i.e., $c_A^H < c_A^L$). Once suit is filed, the high-effort agent finds it relatively cheap to mount a defense, and therefore presents more evidence (and wins more often) than does the low-effort agent.

3.2 Filing Stage

With the equilibrium litigation levels in hand, we now step backwards to analyze the principal's filing decision. Should the principal sue, she expects to receive the payoffs from litigation stage described above, but must pay the fixed costs F of bringing an action. Consequently, the principal will sue only if the former exceeds the latter. (Because the principal's beliefs are constrained to be sequentially rational, she must still conjecture at this stage that there is an α-probability that the agent had previously given effort).

If the principal chooses not to litigate, she simply pays the agent the contracted wage, and thus her low-state payoff is:

$$\pi_P(NS;\alpha) = V_L - w. \tag{9}$$

Conversely, if the principal files suit, her expected payoff is:

$$\pi_P(S;\alpha) = V_L - w + [\alpha \cdot q_H^* + (1-\alpha) \cdot q_L^*] \cdot D - c_P L_P^* - F. \tag{10}$$

Let $R_P(\alpha) \equiv \pi_P(S;\alpha) - \pi_P(NS;\alpha)$ denote the *net gain* the principal expects to receive from suing over abstaining. Clearly, the principal will always abstain from litigating (*i.e.*, set $\gamma = 0$) if $R_P(\alpha) < 0$, and will always file suit (*i.e.*, set $\gamma = 1$) if $R_P(\alpha) > 0$. When $R_P(\alpha) = 0$, however, the principal is indifferent, and would be willing to adopt any $\gamma \in [0,1]$. Using the reduced-form litigation strategies specified above, it is possible to derive the following lemma about the principal's filing decision (whose proof can be found in the appendix):

Lemma 5 *The principal's net expected gain from filing suit, $R_P(\alpha)$, is continuous and strictly decreasing in b, strictly decreasing in α in regions B_4, B_3, B_2, and flat in α in region B_1. Moreover, holding α fixed, $R_P(\alpha)$ is strictly increasing in D, and strictly decreasing in c_P and F.*

The fact that $R_P(\alpha)$ decreases in α is not surprising. Indeed, a marginal increase in α implies that the principal believes it more likely that the agent had previously expended effort. Because high-effort agents are more effective litigators than are their shirking counterparts, one would expect the principal's net expected benefits from filing suit to decrease (as the lemma confirms). From similar logic, as the stakes involved in the suit (D) increase, the principal's incentive to sue is analogously enhanced; but, as either the filing fees (F), the marginal cost of litigation (c_P), or the court's pro-agent bias (b) increase, suit becomes less attractive to the principal.

3.3 Effort Stage

Consider now the agent's *ex ante* effort choice, anticipating the subsequent equilibrium behavior characterized above. Recall that γ denotes the probability that the principal litigates. Accordingly, the agent's expected payoff from high effort is:

$$\pi_A(e^H;\gamma) = w - \gamma(1-p)(q_H^* D + c_A^H L_A^{H^*}) - \phi. \tag{11}$$

Conversely, the agent's expected payoff from low effort is:

$$\pi_A(e^L;\gamma) = w - \gamma p(q_L^* D + c_A^L L_A^{L^*}). \tag{12}$$

In what follows, let $R_A(\gamma) \equiv \pi_A(e^H;\gamma) - \pi_A(e^L;\gamma)$ denote the net gain the agent expects from expending higher effort. The agent always shirks if $R_A(\gamma) < 0$, and always expends effort if $R_A(\gamma) > 0$. When $R_A(\gamma) = 0$, the agent will be indifferent, and thus willing to mix over high and low effort levels. From the reduced forms specified above, it is possible to derive the following lemma about the agent's effort decision:

Lemma 6 *The agent's net expected gain from expending productive effort, $R_A(\gamma)$, is continuous and strictly increasing in γ $\forall\gamma \in [0,1]$. Moreover, holding α and γ fixed, $R_A(\gamma)$ is strictly increasing in D and p, and strictly decreasing in ϕ.*

Similar to the arguments above, an increase in the value γ implies that the agent becomes increasingly convinced of the principal's threat to file suit should a low state obtain. Because suit involves both the prospect of damages and litigation costs (which are higher on the margin for a shirking agent), the agent has a greater incentive to expend effort, which both minimizes the likelihood of a low state and enhances the agent's ability to defend against suit. Analogously, the agent's incentive to expend effort increase with the stakes involved in the suit (D) and the importance of the agent's effort (p). On the other hand, as the immediate cost of effort (ϕ) increases, high effort becomes less attractive to the agent.

3.4 Equilibrium

As noted above, we employ the notion of sequential equilibrium to predict rational play of the game. Having computed the equilibrium litigation levels of all player types (i.e., L_P^*, $L_A^{L^*}$, $L_A^{H\,*}$), all that remains is to specify behavior strategies (β^*, γ^*) implied by the expressions above, and a belief structure for the principal (α^*) that is consistent and sequentially rational. Because each of the principal's relevant information sets is reached with positive probability in this game, consistency is trivially established. Regarding sequential rationality, Bayes' Rule requires that the agent's behavior strategy (β) and the principal's beliefs (α) be related as follows[18]:

$$\alpha = \frac{(1-p)\beta}{(1-p)\beta + p(1-\beta)}$$

[18]Or equivalently, $\beta = \frac{p\alpha}{p\alpha + (1-p)(1-\alpha)}$.

Rather than articulating all of the equilibria that can emerge from this model, it is perhaps more instructive to consider a subset of the parameter space that manifests the principal qualitative equilibrium characteristics that we observe throughout.[19] As noted above, we are particularly interested in those cases for which the underlying presumption can have both over- and under-inclusive tendencies (i.e., false negatives and false positives). Accordingly, we shall hereinafter restrict our equilibrium analysis to parameter values satisfying the following two assumptions.

Assumption 1 $F \leq \frac{D}{2} \cdot \frac{c_A^H}{c_A^L}$.

Assumption 2 $\phi \leq (2p-1) \cdot D \cdot \left(\frac{c_A^H}{c_A^L}\right)^{\frac{3}{2}} \left[2 - \left(\frac{c_A^H}{c_A^L}\right)^{\frac{3}{2}}\right]$

Assumption 1 requires that the fixed costs of filing (F) be small enough to ensure that the principal has the incentive to file suit in the event of a low state (at least with a relatively pro-plaintiff presumption). Analogously, Assumption 2 requires that agent's cost of productive effort (ϕ) be sufficiently small (under a relatively pro-plaintiff presumption) to make effort worthwhile if the agent knows that litigation is certain in a bad state ($\gamma = 1$). Note that both assumptions are satisfied whenever the applicable amount of damages (D) is sufficiently large.[20]

Under these assumptions, the equilibria of the model fall conveniently into the three regions pictured below, corresponding to *strong pro-plaintiff* presumptions (Region I), *intermediate* presumptions (Region II), and *strong pro-defendant* presumptions (Region III), respectively.

Region I: *Strong Pro-Plaintiff Presumptions*	**Region II:** *Intermediate Presumptions*	**Region III:** *Strong Pro-Defendant Presumptions*
$b \leq \underline{b} \equiv \frac{D}{4F} \cdot \frac{c_A^H}{c_P}$	$\underline{b} < b < \overline{b}$	$b \geq \overline{b} \equiv \frac{D}{4F} \cdot \frac{c_A^L}{c_P}$

[19]For readers interested in a full description of the equilibria please consult the Appendix.

[20]More generally, a sufficiently large value of D is necessary for our problem to be an interesting one. For example, consider the extreme case where $F < D$, so that damages cannot even cover the fixed costs of filing. Here, Region III is the only viable region, and the unique equilibrium involves shirking by the agent and abstention by the prinipal. The legal presumption is irrelevant. Alternatively, consider the case where $\phi < (2p-1) \cdot D$, so that damages are so small as to have no deterrent effect on the agent (even for the most potent pro-principal rule). Once again, in such a situation the agent will always choose to shirk regardless of the evidentiary presumption b. (Our Assumptions 1 and 2 are slightly more restrictive than these for expositional reasons, as they ensure that any corner solutions of the litigaiton game occur only for relatively "extreme" presumptions in Region I—i.e., that Region II and Region III are wholly within interval B_4).

The boundaries of these regions correspond to the critical values of b at which the presumption has a dispositive effect on the principal's litigation strategy—*i.e.*, in Region I, the pro-plaintiff presumption is so strong that the principal would *always* sue, regardless of her beliefs about the agent's prior behavior; in Region III, in contrast, the pro-defendant presumption is so strong that the principal would *never* sue, regardless of her equilibrium beliefs; finally, within Region II the underlying presumption is relatively moderate, so that the principal's litigation strategy turns critically on her beliefs about the agent's prior behavior. So long as Assumptions 1 and 2 are satisfied, all three regions described in the table above contain unique sequential equilibria. We address each region below (albeit in a slightly inverted order).

Strong Pro-Defendant Presumptions: $b \in [\overline{b}, \infty)$. In Region III, the agent benefits from a pro-defendant presumption that is sufficiently strong to deter the principal altogether from filing suit, regardless of her equilibrium beliefs. Thus, any equilibrium in this region must prescribe that the principal employs a pure strategy of abstaining from suing. Without fear of suit, one would also expect the agent to pursue a pure strategy of shirking, a result embodied in the following proposition[21]:

Proposition 1 *If* $b \in [\overline{b}, \infty)$, *then there exists a unique sequential equilibrium in pure strategies with* $\beta^* = 0$, $\gamma^* = 0$, *and* $\alpha^* = 0$. *The equilibrium litigation levels* L_P^*, $L_A^{L^*}$ *and* $L_A^{H\,*}$ *are given in the first row of the table from Lemma 1.*

Proposition 1 is in many ways quite intuitive. It states that if the pro-agent legal presumption, b, grows sufficiently large, the prospects from suit are not enough to cover the principal's fixed cost of filing, F. Consequently, the principal poses no credible threat to file suit. Knowing this, the agent is undeterred from shirking, and therefore always expends low levels of effort.[22] The social cost of this equilibrium, then, consists solely of the costs imposed by sub-optimal effort.

Strong Pro-Plaintiff Presumptions: $b \in [0, \underline{b}]$. Consider now the opposite case in Region I, where a court adheres to a strong pro-plaintiff presumption. Here, the principal has such a clear upper hand in litigation that she always files suit regardless of her beliefs about the agent type she faces. So long as damages impose a sufficiently strong deterrent effect (as embodied by Assumption 1), one can show

[21]This proposition is simply stated without proof.

[22]Note also that if a court were free to choose damages as well, D, it could effect the same outcome by specifying small or zero damages (and thus Region III would effectively span all of b-space). Equivalently, this proposition does not depend on either Assumption 1 or Assumption 2.

that the agent also follows a pure strategy of expending effort. This argument is illustrated in the following proposition:[23]

Proposition 2: *If $b \in [0, \underline{b}]$ and Assumptions 1 and 2 hold, then there exists a unique sequential equilibrium in pure strategies with $\beta^* = 1$, $\gamma^* = 1$, and $\alpha^* = 1$. The equilibrium litigation strategies are given by Lemma 1, and depend on the precise value of b.*

So long as effort is socially desirable, the equilibrium social waste in Region I consists only of the costs due to litigation. An important consideration within this subregion is the limiting case where $b = 0$. Here, the pro-principal presumption is sufficiently inviolate that agent can never prevail in litigation, and thus essentially the principal need only pay the filing fee F to collect damages. Thus, $b = 0$ reflects a form of strict liability rule favoring the plaintiff. Perhaps more illustratively, such a rule is the doctrinal equivalent of an ordinary incentive contract, paying the agent w in the high state and $w - D$ in the low state (though one that costs the principal F to invoke should the low state obtain).

Intermediate Presumptions: $b \in (\underline{b}, \overline{b})$. Finally, consider what is perhaps the most observationally familiar region, in which the legal presumption is not preclusive in equilibrium, and thus each side has a legitimate prospect of prevailing. Region II is also the most interesting from a game-theoretic perspective, because it entails equilibria in mixed-strategies, and thus the agent randomizes between working hard and shirking, while the principal randomizes between suit and abstention. This statement is formalized in the following proposition:[24]

Proposition 3 *If $b \in (\underline{b}, \overline{b})$ and Assumptions 1 and 2 hold, then there exists a unique equilibrium in mixed strategies with $\beta^* \in (0,1)$, $\gamma^* \in (0,1)$ and $\alpha^* \in (0,1)$. The equilibrium litigation levels L_P^*, $L_A^{L^*}$ and $L_A^{H\,*}$ are given in the first row of the table from Lemma 1.*

That mixed strategy equilibria characterize Region II should not be terribly surprising, as it is a common result in asymmetric information models of auditing. To understand the core intuition, consider the agent's best response in this region if he conjectured that the principal would never sue. Undeterred by the spectre of legal action, the agent would never give effort. In response, however, the principal would always sue, which in turn would induce the agent to work hard rather than

[23]The proof of this proposition appears in the Appendix.

[24]The proof of this proposition appears in the Appendix.

shirk. This cycling iteration of best responses implies that the only equilibrium in Region II must be in mixed strategies, as stated in the proposition.

Perhaps of greater interest in this subregion are the comparative statics on the players' equilibrium effort and suit strategies, (β^* and γ^*) as b changes. Implicit differentiation of the principal's and agent's best response functions[25] yields the following:

Proposition 4 *If $b \in (\underline{b}, \overline{b})$ and Assumptions 1 and 2 hold, the agent's equilibrium effort strategy β^* is strictly decreasing in b, and the principal's equilibrium filing strategy γ^* is strictly increasing in b for all $p \in [\hat{p}, 1]$* where $\hat{p} \in [\frac{1}{2}, 1)$.

The intuition behind this Proposition is as follows. Consider first the marginal impact of increasing b on the agent's equilibrium effort choice β^*. As the agent's power increases, he becomes increasingly effective at fending off litigation—a source of confidence that leads him to shirk more often in equilibrium. One might similarly conjecture that increasing b would have the opposite effect on the principal's filing strategy—*i.e.,* facing a presumption that is slightly more biased in favor of the agent, the principal would be less likely to file suit in the event of a bad state. Surprisingly, however, this is not what we find. Rather, the above proposition states that increasing b actually *enhances* principal's proclivity to litigate in the low state. On first blush, this is a surprising result: all else equal, larger values of b should reduce the principal's expected payoff from filing. This reasoning, however, fails to account for the fact that *in equilibrium,* a larger b also induces the agent to reduce his effort in the primary activity. Knowing this, the principal is more confident that the agent's shirking has contributed to the realization of a low state, which increases her incentive to sue. When Assumptions 1 and 2 are satisfied, this indirect equilibrium more than offsets the direct incentive effect, thereby leading to a a greater likelihood of suit when a low state occurs.

A number of corollaries are direct implications of Proposition 4. Three of them, however, deserve particular attention:

Corollary 4.1 *If $b \in (\underline{b}, \overline{b})$ and Assumptions 1 and 2 hold, the likelihood of a false positive (high-effort defendant is found liable, i.e. q_H^*) is decreasing in b.*

Corollary 4.2 *If $b \in (\underline{b}, \overline{b})$ and Assumptions 1 and 2 hold, the likelihood of a false negative (low-effort defendant is not found liable, i.e. $1 - q_L^*$) is increasing in b.*

[25] Because the unique sequential equilibrium is in mixed strategies, it is characterized by the equations $R_P(\alpha^*) = 0$ and $R_A(\gamma^*) = 0$, embodying indifference expressions of the principal's and agent's best-response functions. Proposition _ reports comparative statics on this system.

Corollary 4.3 *If $b \in (\underline{b}, \overline{b})$ and Assumptions 1 and 2 hold, the ex ante probability of suit is increasing in b for sufficiently large p.*

Corollary 4.4 *If $b \in (\underline{b}, \overline{b})$ and Assumptions 1 and 2 hold, the unconditional probability plaintiff victory at trial does not vary with b.*

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5 Appendix:

5.1 Litigation Effort Lemma

Lemma 1 *The equilibrium litigation efforts of the principal and the two agent types in b−space (along with the equilibrium probabilities of liability) are:*

Interval	L_P^*	L_A^{H*}	L_A^{L*}	q_H^*	q_L^*
$b \in B_4$	$\frac{D \cdot \overline{\mu}^2}{4b}$	$\frac{D \cdot \overline{\mu}}{2b} \cdot \left[\frac{1}{\sqrt{c_A^H}} - \frac{\overline{\mu}}{2b} \right]$	$\frac{D \cdot \overline{\mu}}{2b} \cdot \left[\frac{1}{\sqrt{c_A^L}} - \frac{\overline{\mu}}{2b} \right]$	$\frac{\overline{\mu}\sqrt{c_A^H}}{2b}$	$\frac{\overline{\mu}\sqrt{c_A^L}}{2b}$
$b \in B_3$	$\frac{b \cdot D}{c_A^L}$	$\frac{D}{\sqrt{c_A^L}} \cdot \left[\frac{1}{\sqrt{c_A^H}} - \frac{1}{\sqrt{c_A^L}} \right]$	0	$\sqrt{\frac{c_A^H}{c_A^L}}$	1
$b \in B_2$	$\frac{D \cdot \underline{\mu}^2}{4b}$	$\frac{D \cdot \underline{\mu}}{2b} \cdot \left[\frac{1}{\sqrt{c_A^H}} - \frac{\underline{\mu}}{2b} \right]$	0	$\frac{\underline{\mu}\sqrt{c_A^H}}{2b}$	1
$b \in B_1$	$\frac{b \cdot D}{c_A^H}$	0	0	1	1

where $\underline{\mu} \equiv \left(\frac{\alpha\sqrt{c_A^H}}{c_P} \right)$ *and* $\overline{\mu} \equiv \left(\frac{\alpha\sqrt{c_A^H} + (1-\alpha)\sqrt{c_A^L}}{c_P} \right)$, *and* $B_1 \equiv \left[0, \frac{1}{2}\underline{\mu}\sqrt{c_A^H} \right)$, $B_2 \equiv \left[\frac{1}{2}\underline{\mu}\sqrt{c_A^H}, \frac{1}{2}\underline{\mu}\sqrt{c_A^L} \right)$, $B_3 \equiv \left[\frac{1}{2}\underline{\mu}\sqrt{c_A^L}, \frac{1}{2}\overline{\mu}\sqrt{c_A^L} \right)$, *and* $B_4 \equiv \left[\frac{1}{2}\overline{\mu}\sqrt{c_A^L}, \infty \right)$.

Proof: In interval B_4, the equilibrium strategies are found simply by solving the players' first order conditions (8), (4), and (6) simultaneously. It is easily confirmed within this interval that the proposed strategies constitute an interior solution for all player-types.

Outside of interval B_4, however, the low-effort agent's non-negativity constraint binds, and the solution proposed is not valid. Any such solution must place the low-effort agent at a corner solution of $L_A^{L*} = 0$. In selecting her strategy, though, the principal may not be able simply to ignore the low-effort agent and optimize against the high agent. Indeed, if the principal adjusts her strategy to focus solely on the high-effort agent, the low-effort agent may enter the fray again. For the moment, however, assume that the principal takes for granted that she'll prevail against the low-effort agent, and simply chooses L_P to optimize against the α-probability of a high-effort opponent. In such a circumstance, one can solve the principal's first-order condition to yield:

$$L_P^* \quad = \quad \frac{D}{4b} \cdot \left(\underline{\mu} \right)^2 \tag{13}$$

where $\underline{\mu} \equiv \left(\frac{\alpha\sqrt{c_A^H}}{c_P} \right) = \overline{\mu} - \frac{(1-\alpha)\sqrt{c_A^H}}{c_P} < \overline{\mu}$. (Under this proposed solution, it is easily confirmed that the high-agent agent type strategy is interior so long as $b \geq \frac{\underline{\mu}}{2}\sqrt{c_A^H}$.
)

We still must check, however, whether the low-type agent would indeed be at a corner solution when the principal expends the above proposed litigation level. Equivalently we must determine whether:

$$\left(\frac{b\frac{D}{4b} \cdot \left(\underline{\mu}\right)^2}{\left(b \cdot 0 + \frac{D}{4b} \cdot \left(\underline{\mu}\right)^2\right)^2} \right) \cdot D \leq c_A^L, \tag{14}$$

a condition which is satisfied only if $b \leq \frac{\underline{\mu}}{2}\sqrt{c_A^L} < \frac{\overline{\mu}}{2}\sqrt{c_A^L}$. Thus, this solution described in (13) describes an equilibrium only for $b \in \left[\frac{\underline{\mu}}{2}\sqrt{c_A^H}, \frac{\underline{\mu}}{2}\sqrt{c_A^L}\right) \equiv B_2$. Importantly, it is *not* applicable when $b \in \left[\frac{\underline{\mu}}{2}\sqrt{c_A^L}, \frac{\overline{\mu}}{2}\sqrt{c_A^L}\right) \equiv B_3$, a region in which the principal's optimal choice is always at a "kink" in her indirect payoff function. To solve for the location of this kink, one must determine the lowest litigation level by the principal that will make the low-effort agent's non-negativity constraint binding. Equivalently, L_P^* must satisfy:

$$\left(\frac{bL_P^*}{(b \cdot 0 + L_P^*)^2} \right) \cdot D = c_A^L, \tag{15}$$

which implies:

$$L_P^* = \left(\frac{b \cdot D}{c_A^L} \right). \tag{16}$$

Using similar logic, it is possible to summarize the players' strategies in interval B_1. *Q.E.D.*

5.2 Filing Stage Lemma

Lemma 5: The principal's net expected gain from filing suit, $R_P(\alpha)$, is continuous and strictly decreasing in b, strictly decreasing in α in regions B_4, B_3, B_2, and flat in α in region B_1. Moreover, $R_P(\alpha)$ is strictly increasing in D, and strictly decreasing in c_P and F.

Proof: From the equilibrium probabilities and strategies derived in the text, the expected indirect payoff functions of the players, conditional on litigation, are illustrated in the table below. (Note that the two agent-types indirect payoff functions are in actuality expected losses; they are therefore expressed in the negative.)

Interval	$\pi_P\left(L_P^*\right)$	$-\pi_A^H(L_A^{H*})$	$-\pi_A^L(L_A^{L*})$
$b \in B_4$	$\frac{D \cdot \overline{\mu}^2 \cdot c_P}{4b}$	$\frac{D \cdot \overline{\mu}}{b}\left[\sqrt{c_A^H} - \frac{\overline{\mu} \cdot c_A^H}{4b}\right]$	$\frac{D \cdot \overline{\mu}}{b}\left[\sqrt{c_A^L} - \frac{\overline{\mu} \cdot c_A^L}{4b}\right]$
$b \in B_3$	$\frac{D \cdot c_P}{c_A^L} \cdot \left(\underline{\mu}\sqrt{c_A^L} - b\right) + (1-\alpha)D$	$D\sqrt{\frac{c_A^H}{c_A^L}}\left[2 - \sqrt{\frac{c_A^H}{c_A^L}}\right]$	D
$b \in B_2$	$\frac{D \cdot \underline{\mu}^2 \cdot c_P}{4b} + (1-\alpha)D$	$\frac{D \cdot \underline{\mu}}{b} \cdot \left[\sqrt{c_A^H} - \frac{\underline{\mu} \cdot c_A^H}{4b}\right]$	D
$b \in B_1$	$D \cdot \left(1 - \frac{b \cdot c_P}{c_A^H}\right)$	D	D

Since $R_P(\alpha) \equiv \pi_P\left(L_P^*(\alpha)\right) - F$, piecewise differentiation immediately yields the results.

5.3 Effort Stage Lemma

Lemma 6: The agent's net expected gain from expending productive effort, $R_A(\gamma)$, is continuous and strictly increasing in γ $\forall \gamma \in [0,1]$. Moreover, $R_A(\gamma)$ is strictly increasing in D and p, and strictly decreasing in ϕ.

Proof: By definition $R_A(\gamma) \equiv \gamma\left[(1-p)\pi_A^H(L_A^{H*}) - p\pi_A^L(L_A^{L*})\right] - \phi$ where $\pi_A^H(L_A^{H*})$ and $\pi_A^L(L_A^{L*})$ are defined in the proof of Lemma 5. Examining each of the four subregions separately and differentiating yields the desired result.

5.4 Equilibrium Propositions

Proposition 2 *If $b \in [0, \underline{b}]$ and Assumptions 1 and 2 hold, then there exists a unique sequential equilibrium in pure strategies with $\beta^* = 1$, $\gamma^* = 1$, and $\alpha^* = 1$. The equilibrium litigation strategies correspond to those in the table from Lemma 1, and depend on the precise value of b.*

Proof: The proof of the proposition consists of an iterated dominance argument. First, we show that when $b \in [0, \underline{b}]$, it is strictly dominant for the principal to file suit ($\gamma = 1$). Then, we show that when $b \in [0, \underline{b}]$, expending effort (*i.e.*, $\beta = \alpha = 1$) is the agent's unique best response to the principal's dominant strategy $\gamma = 1$. We construct the proof using two lemmas:

Lemma A *Assumption 1 implies that the principal will always file suit when $b \in [0, \underline{b}]$.*

Proof: From Lemma 5 we know that $R_P(\alpha)$ is decreasing in α in regions B2, B3, and B4 and flat in α for region B1 thus it is sufficient to show that $R_P(1) \geq 0$ for each of the four regions. Note, however, that if $\alpha = 1$ then region B3 is empty.

In Region B4, $R_P(1) = \frac{Dc_A^H}{4bc_P} - F \geq 0$ since $b \leq \frac{Dc_A^H}{4Fc_P}$. Clearly, in Region B2, $R_P(1) = \frac{Dc_A^H}{4bc_P} + (1-\alpha)D - F \geq 0$. Finally, in Region B1 we have $R_P(1) = D(1 - \frac{bc_P}{c_A^H}) - F \geq D(1 - \frac{D}{4F}) - F \geq 0$ if $D \geq 2F$ which is guaranteed by Assumption 1.

Lemma B *Assumptions 1 and 2 imply that the agent's unique best response to $\gamma = 1$ is to expend productive effort when $b \in [0, \overline{b}]$.*

Proof: First, note that if effort is the agent's unique best response on the interval $b \in [0, \overline{b}]$, then it must also be the unique best response for $b \in [0, \underline{b}] \subset [0, \overline{b}]$. (We focus on the more restrictive case for purposes of later propositions). Begin once again by concentrating solely on the interval B_4 as defined in Lemma 1. Denote $b_4(\alpha) < \underline{b} < \overline{b}$ as the lower boundary of B_4. We need to show that if $b \in \left[b_4(\alpha), \overline{b}\right]$, then $R_A(1) > 0$.

Thus, suppose that $b < \overline{b}$ and thus the agent knows the principal will sue ($\gamma = 1$). The agent will expend effort if and only if:

$$\phi \leq \frac{D \cdot \overline{\mu}}{b} \cdot \left[p \cdot \left(\sqrt{c_A^L} - \frac{\overline{\mu} c_A^L}{4b} \right) - (1-p) \cdot \left(\sqrt{c_A^H} - \frac{\overline{\mu} c_A^H}{4b} \right) \right],$$

But since a high-effort agent suffers less in litigation than does a low-effort agent, the above condition is always satisfied if the following sufficient condition is satisfied:

$$\phi \leq (2p-1) \cdot D \cdot \left[\frac{\overline{\mu}}{b} \sqrt{c_A^L} - \frac{\overline{\mu}^2}{4b^2} c_A^L \right] \equiv (2p-1) \cdot D \cdot [\Psi(\alpha)].$$

Let the critical value ϕ^c that be the value of ϕ that satisfies this expression for every possible value of α. To compute ϕ^c, we need to compute the value of α and b that minimizes $\Psi(\alpha)$. To do this, note the following:

1. $\Psi(\alpha)$ is decreasing in $b \in [b_4(\alpha), \overline{b}]$ for all values of α. To see this, simply take the derivative of $\Psi(\alpha)$ with respect to b:

$$\begin{aligned} \frac{\partial \Psi(\alpha)}{\partial b} &= \frac{\partial}{\partial b} \left[\frac{\overline{\mu}}{b} \sqrt{c_A^L} - \frac{\overline{\mu}^2}{4b^2} c_A^L \right] \\ &= \overline{\mu} \sqrt{c_A^L} \frac{1}{b^3} \left(\frac{\overline{\mu}}{2} \sqrt{c_A^L} - b \right) \\ &= \overline{\mu} \sqrt{c_A^L} \frac{1}{b^3} (b_4(\alpha) - b) < 0 \end{aligned}$$

   Thus, it is clear that $\Psi(\alpha)$ must reach its minimum value at $b = \overline{b}$ for the relevant region.

2. The minimum value of $\Psi(\alpha)$ evaluated at $b = \overline{b}$ must occur at $\alpha = 1$ which follows from differentiating w.r.t. α and using Assumption 1.

3. Evaluating $\Psi(1)$ at $b = \overline{b}$ yields:

$$
\begin{aligned}
\phi &\leq (2p-1)\cdot D \cdot \left[\Psi(1)|_{b=\overline{b}}\right] \\
\phi &\leq (2p-1)\cdot D \cdot \frac{\overline{\mu}}{\overline{b}}\left[\sqrt{c_A^L} - \frac{\overline{\mu}}{4\overline{b}} c_A^L\right] \\
\phi &\leq \phi^c \equiv (2p-1)\cdot 4F\sqrt{\frac{c_A^H}{c_A^L}}\left[1 - \frac{F}{D}\sqrt{\frac{c_A^H}{c_A^L}}\right]
\end{aligned}
$$

Now, using Assumption 1, it is clear that the sharpest possible bound imposed by ϕ^c is found when $F = \frac{D}{2}\left(\frac{c_A^H}{c_A^L}\right)$. Thus, imposing this condition on the above yields:

$$\phi \leq (2p-1)\cdot D \cdot \frac{c_A^H}{c_A^L}\cdot\sqrt{\frac{c_A^H}{c_A^L}}\left[2 - \frac{c_A^H}{c_A^L}\cdot\sqrt{\frac{c_A^H}{c_A^L}}\right] \leq (2p-1)D$$

which is the condition given in Assumption 2.

In all other regions (B_1, B_2, B_3), it is significantly simpler to show that suit by the principal makes it strictly dominant for the agent to expend effort. Indeed, in these regions if the principal sues, a sufficient condition for the agent to expend effort is:

$$\phi \leq (2p-1)\cdot D$$

But this condition is implied the condition $\phi \leq \phi^c$, and thus for all other Regions, the agent's best response to $\gamma = 1$ is to expend effort.

Proposition 3 *If $b \in (\underline{b}, \overline{b})$ and Assumptions 1 and 2 hold, then there exists a unique equilibrium in mixed strategies with $\beta^* \in (0,1), \gamma^* \in (0,1)$ and beliefs $\alpha^* \in (0,1)$. The equilibrium litigation levels L_P^*, $L_A^{L^*}$ and $L_A^{H\,*}$ correspond to those in the first row of the table from Lemma 1.*

Proof: We prove the proposition first by showing that there are no pure strategy equilibria in Region II, and then by showing that the set of mixed strategy is a singleton. We because there is a one-to-one mapping between α and β, we will describe the equilibria solely in terms of α and γ.

Note first that there can never be an equilibrium in Region II that involves the pure strategy $\gamma = 0$. Suppose there were; knowing that the principal would never sue, the agent would always shirk ($\alpha = 0$). However, in Region II, the principal's best response to shirking by the agent is to sue always, which is a contradiction. Similarly, there can be no Region II equilibrium that involves the pure strategy $\gamma = 1$. Lemma B above ensures that the agent's best response in Region II to suit by the principal is to work hard ($\alpha = 1$); but in Region II $b > \underline{b}$, and thus the principal's best response to beliefs ($\alpha = 1$) is not to sue, which is a contradiction. An identical argument establishes that there can be no pure strategy equilibrium in Region II involving $\beta = \alpha = 0$ or $\beta = \alpha = 1$.

Given that there are no pure-strategy equilibria in Region II, we need only verify that the system

$$\begin{aligned} R_A(\gamma) &\equiv -\phi + \gamma \cdot \left[p(q_L^* D + c_A^L L_A^{L^*}) - (1-p)(q_H^* D + c_A^H L_A^{H^*}) \right] = 0 \\ R_P(\alpha) &\equiv -F + [\alpha \cdot q_H^* + (1-\alpha) \cdot q_L^*] \cdot D - c_P L_P^* = 0 \end{aligned}$$

has a unique interior solution for α and γ. Inserting the functional forms for the indirect utility functions yields:

$$\begin{aligned} -\phi + \gamma \cdot \left[p \left(\frac{D \cdot \overline{\mu}}{b} \left[\sqrt{c_A^L} - \frac{\overline{\mu} \cdot c_A^L}{4b} \right] \right) - (1-p) \left(\frac{D \cdot \overline{\mu}}{b} \left[\sqrt{c_A^H} - \frac{\overline{\mu} \cdot c_A^H}{4b} \right] \right) \right] &= 0 \\ -F + \frac{D \cdot c_P}{4b} (\overline{\mu})^2 &= 0 \end{aligned}$$

Note that the second equation characterizes α uniquely. And, fixing α, $R_A(\gamma)$ is linear in γ, and thus also has a unique solution in γ. Therefore, the equilibrium is unique.

Proposition 4 *If $b \in (\underline{b}, \overline{b})$ and Assumptions 1 and 2 hold, the agent's equilibrium effort strategy β^* is decreasing in b, and the principal's equilibrium filing strategy γ^* is increasing in b for all $p \in [\hat{p}, 1]$* where $\hat{p} \in [\frac{1}{2}, 1)$.

Proof: Solving the indifference condition for the principal yields the equilibrium value of α^*:

$$\alpha^* = \frac{\sqrt{c_A^L} - \sqrt{\frac{4b \cdot F \cdot c_P}{D}}}{\left(\sqrt{c_A^L} - \sqrt{c_A^H} \right)} \tag{17}$$

It is easily confirmed from (17) that $\alpha^* \in (0,1)$ when $b \in (\underline{b}, \overline{b})$, and that α^* is strictly decreasing (from 1 to 0) in b over this interval. Inserting this solution into the agent's indifference condition yields:

$$\gamma^* = \frac{\phi/D}{\sqrt{\frac{F}{Dbc_P}}\left[2\cdot\left(p\sqrt{c_A^L}-(1-p)\sqrt{c_A^H}\right)-\sqrt{\frac{F}{Dbc_P}}\cdot\left(pc_A^L-(1-p)c_A^H\right)\right]} \tag{18}$$

Differentiation of (18) with respect to b directly yields the result reported in the Proposition, that γ^* is increasing in b for sufficiently large p. It is clear that γ^* is increasing in b if and only if the denominator is decreasing in b. We can re-express the denominator as $\kappa\,[2A-\kappa C]$, where $\kappa = \sqrt{\frac{F}{Dbc_P}}$, $A = \left(p\sqrt{c_A^L}-(1-p)\sqrt{c_A^H}\right)$, and $C = \left(pc_A^L-(1-p)c_A^H\right)$. Thus, the denominator is decreasing in b if $2\cdot\left(\frac{\partial\kappa}{\partial b}\right)\cdot[A-\kappa C]<0$. But we know that $\frac{\partial\kappa}{\partial b}<0$, so in order to prove the proposition, we must show that $A>\kappa C$ for large enough p. Equivalently, we must show that:

$$b > \frac{F}{D\cdot c_P}\left(\frac{pc_A^L-(1-p)c_A^H}{p\sqrt{c_A^L}-(1-p)\sqrt{c_A^H}}\right)^2$$

for sufficiently large p. But we know that the term in the brackets is strictly decreasing in p, and thus the above condition is satisfied for all p if it is satisfied for $p=\frac{1}{2}$. Evaluated at $p=\frac{1}{2}$ our condition is $b>\frac{F}{Dc_P}(\sqrt{c_A^L}+\sqrt{c_A^H})^2$ which is not always assured. However, evaluated at $p=1$ our condition is

$$b > \frac{F\cdot c_A^L}{D\cdot c_P}$$

which is assured by Assumption 1. Thus, there exists a $\hat{p}\in[\frac{1}{2},1]$ above which γ^* is increasing in b.

Corollary 4.1 *If $b\in(\underline{b},\overline{b})$ and Assumptions 1 and 2 hold, the likelihood of a false positive (high-effort defendant is found liable, i.e. q_H^*) is decreasing in b.*

Corollary 4.2 *If $b\in(\underline{b},\overline{b})$ and Assumptions 1 and 2 hold, the likelihood of a false negative (low-effort defendant is not found liable, i.e. $1-q_L^*$) is increasing in b.*

Corollary 4.3 *If $b\in(\underline{b},\overline{b})$ and Assumptions 1 and 2 hold, the ex ante probability of suit is increasing in b for sufficiently large p.*

Corollary 4.4 *If $b\in(\underline{b},\overline{b})$ and Assumptions 1 and 2 hold, the unconditional probability plaintiff victory at trial does not vary with b.*

Proofs of Corollaries 4.1 - 4.4: In Region B4 we know $q_H^* = \frac{\bar{\mu}\sqrt{c_A^H}}{2b}$ and $q_L^* = \frac{\bar{\mu}\sqrt{c_A^L}}{2b}$. Note that $\alpha^* = \frac{\sqrt{c_A^L} - \sqrt{\frac{4b \cdot F \cdot c_P}{D}}}{\left(\sqrt{c_A^L} - \sqrt{c_A^H}\right)}$ we have $\bar{\mu} = \sqrt{\frac{4bF}{Dc_P}}$ and $\frac{\delta\bar{\mu}}{\delta b} = \sqrt{\frac{F}{bDc_P}}$. Differentiating q_H^* and q_L^* with respect to b now immediately yields Corollaries 4.1 and 4.2.

Corollary 4.3 follows immediately from Proposition 4. Finally, the unconditional probability of plaintiff victory is given by $\bar{q} = \alpha^* q_H^* + (1-\alpha^*) q_L^* = \frac{\bar{\mu}^2 c_P}{2b}$. Differentiating with respect to b yields $\frac{\delta\bar{q}}{\delta b} = 0$.

Figure 1: Extensive Form of the Game